HELICOPTER GUST RESPONSE AND ROTOR FLAPPING FEEDBACK CONTROL

Department of Aerospace and Mechanical Sciences

Report No. 884

BY

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DEPARTMENT OF AEROSPACE AND MECHANICAL SCIENCES

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Submitted in partial fulfullment of the requirements for the degree of Master of Science in Engineering from Princeton University, 1969.

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ABSTRACT

In Part I the primary longitudinal stability derivatives of a "conventional" airplane and a "typical" large, high speed, single rotor helicopter are simply compared, and fundamental similarities and differences discussed. Physical interpretation and meaning is given to various aspects of the character of the two types of aircraft.

In Part II the influence of rotor flapping (tip path plane orientation and coning) on the stability derivatives of the whole helicopter is investigated, and then various stabilization schemes in which the flapping of the rotor is the basis for the feedback signal are explored. In particular are the effects of Oehmichen and Delta Three pitch-flap couplings on the static stability of the helicopter. It is found that of the techniques investigated and within the limits of validity of this work, only negative Delta Three (positive pitch-flap coupling) is effective in improving the longitudinal gust response characteristics in high speed flight.

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LIST OF SYMBOLS

A	rotor disk area, ft^2
ANF	axis of no feathering (see Appendix II)
a (a_w)	slope of blade (wing) lift curve, per radian
a'	angle of R vector with respect to ANF, radians
a_1	longitudinal blade flapping angle (TPP) with respect to ANF, radians
a_0	collective (coning) blade flapping angle, radians
B_{1S}, ΔB_{1S}	longitudinal cyclic control input, ANF with respect to shaft, radians
b_1	lateral blade flapping angle (TPP) with respect to ANF, radians
b	number of blades
C_H	$H/\rho\pi R^2(\Omega R)^2$
c ($\bar{c}$)	average blade (wing) chord, feet
C_T	$T/\rho\pi R^2(\Omega R)^2$
C_L	L/qs for airplane, $L/q\pi R^2$ for helicopter
e	hinge offset (see Appendix II)
F.S.	fuselage station
g	force of gravity, 32.2 ft/sec^2
H	inplane rotor force, or component of R vector along ANF, lbs.
h	hub height above C.G., feet (see Appendix II)
$\bar{h}$	distance between C.G. and shaft, feet (see Appendix II)
I_1	blade mass moment of inertia about flapping hinge, slug - ft^2
$I_{(\ )}$	moment of inertia about ( ) axis, slug - ft^2
i_t, i_s	tail incidence, shaft incidence, with respect to fuselage radians (see Appendix II)
J	gain of collective feedback

LIST OF SYMBOLS (CONTINUED)

J', J'', J''_δ	see equations II (7) - II (12)
K (K_δ)	gain of cyclic component of Oehmichen (Delta Three) feedback
K', K'_δ, K'', K''_δ	see equations II (7) - II (12)
K_y	radius of gyration, feet
L	lift, lbs.
l_t	distance from C.G. to horizontal tail, feet
M	moments about Y axis, ft-lb.
$M_{(\ )}$	moment stability derivative, $\frac{1}{m}\frac{\partial M}{\partial (\ )}$
M_s	blade mass moment = $m\int_e^R (r-e)\,dr$, slug-ft.^{2}
m	mass; running mass of blade, slug - ft
n_z	normal acceleration, normal load; n times force of gravity along Z axis, "g's"
q	dynamic pressure, lb/ft^2
R	rotor resultant force; rotor radius
s	Laplace transform variable, $^1/$sec
S	area (wing, rotor disk) ft^2
T	thrust, lbs.
TPP	"plane" formed by path of blade tips around rotor azimuth
U_o, V_o	forward trim, or initial, velocity, ft/sec. or kts.
u, Δu	perturbation velocity in X direction, ft/sec.
v_i	rotor induced velocity, ft/sec.
V_t	tail "volume"

LIST OF SYMBOLS (CONTINUED)

W.L.	"water-line"; horizontal fuselage reference
W	gross weight, lbs.
w, Δw	perturbation velocity in Z direction, ft/sec.
$X_{(\ )}$	horizontal stability derivative, $\frac{1}{m}\frac{\partial X}{\partial (\ )}$
$Z_{(\ )}$	vertical stability derivative $\frac{1}{m}\frac{\partial Z}{\partial (\ )}$
X,Y,Z	forces along X,Y,Z axes (see Appendix II), lbs.
α, $\Delta\alpha$	perturbation angle of attack, radians
β	blade flapping angle, radians
γ	Locke Number = $\rho a c R^4 / I_1$
δ	Delta Three hinge angle with respect to shaft
Δ	increment
ϵ	orientation of R vector in trim = $\left[a' - (B_{1s} + i_s) + \bar{h}/h\right]$
σ	solidity, $bc/\pi R$
η	"efficiency" factor, q local/q
θ, $\Delta\theta$	perturbation pitch angle, radians
θ_0, $\Delta\theta_0$	collective control input, radians
λ	inflow ratio, $[V \cos\alpha - v_i]/\Omega R$
μ	airplane "relative density" = $m/\rho S \bar{c}$, tip speed ratio, $V_0/\Omega R$
ρ	air density, slugs/ft^3
$\tau = {}^1/T$	time constant 1/sec
Ψ	rotor blade azimuth position, ψ = 0 downwind
Ω	rotor angular velocity rad/sec
$(\ )_a$	aircraft
$(\ )_g$	gust
$(\ )_f$	fuselage
$(\ )_t$	tail
$(\ )_0$	initial, trim

LIST OF SYMBOLS (CONTINUED)

$(\ )\Big\vert_{[\ ]}$	[ ] held constant
$(\ )\,A/_{P}$	"auto-pilot", S.A.S.
$(\ )_x,\ (\ )_y$	referred to space axes
$(\dot{\ }),\ (\ddot{\ })$	time derivatives,
$(\ )'$	stability derivative in ( ) is modified by feedback

INTRODUCTION

This thesis is in two parts. The first is a rudimentary comparison of the longitudinal gust response characteristics of a "conventional" airplane with a "typical" large, high speed, single rotor helicopter. The second part consists of the investigation of various forms of helicopter stabilization techniques, especially those using rotor flapping as the sensed element. These two subjects, while related, are themselves topics of considerable reach, and their being treated together in a master's degree thesis is indicative of the depth they are explored. There is a reasoning, however, behind this overview type of approach.

Helicopter pilots have always complained about the so called flying qualities of their craft in gusty weather. "They blow around too much." In order to find out why they do so, and in what way, it is expedient to relate the problem to one whose characteristics are better known - the airplane. The problem is commonly formulated as a set of differential equations with constant coefficients, and the characteristics of the problem are the signs and magnitudes of the coefficients - the stability derivatives. Trends, order of magnitudes, and signs of these derivatives indicate fundamental differences and similarities between the two types of aircraft and lead to insight as to why the helicopter responds more severely to gusts. The rudimentary form of the analysis is such that only these types of conclusion can, and are, drawn.

The first part is thus a review of the derivation of (what are considered to be) the important stability derivatives for aircraft gust response, and the reasoning behind several approximations to the full equations of motion with the author's two cents worth of physical interpretation and understanding added. It is, so to speak, the statement of the problem to be solved in the second part.

In the second part, again, there is a good deal of review, the emphasis being on the physical interpretation and consequences of the findings. The influence of rotor flapping on the stability derivatives of the whole helicopter, and hence the equations that govern how the pilot will be bounced around, is investigated. The impetus for this is to try to discover, in a simple, fundamental manner, whether or not stabilization techniques involving the reduction of rotor flapping as a means to improve helicopter flying qualities are as "good" as they might seem from a more cursory glance at the problem. This is done by first deleting the effects of flapping in the expressions for the stability derivatives, and then by investigating feedback techniques such as Oehmichen and Delta Three pitch-flap couplings. Much time is spent with these two schemes because, as is shown, they form the basis for several other stabilizer techniques which have been proposed. Finally, all of these results are contrasted with a conventional body mounted rate and attitude SAS (stability augmentation system).

All of the analytical analyses are complimented by dynamic analog computer responses. The data which is used to generate the helicopter models is presented in Appendix I, and serves to characterize the large, high speed, single rotor helicopter. Additionally, in the appendices are given a treatment of axis system conversion, and the definition of gust alleviation as it is used in this work.

DESCRIPTION OF ANALYSIS

The equations of motion used as a model of the aircraft in this work are the standard, small perturbation equations commonly used in stability analyses. They are extremely simplified by the approximations and assumptions that form the basis for this study - and result in a form of equations in which only general trends and important characteristics can be discovered. The justification for all of the simplification is an interest in finding order of magnitude trends that are characteristic of fixed or rotating wing aircraft, rather than the details that may vary considerably among aircraft of the same type. Some of the assumptions are purely for simplicity and others are assumptions that have been shown to be valid for an analysis of this sort, provided certain restrictions are kept in mind. In particular are the effects of high gain feedback designs involving the rotor in the feedback loop. In such cases known stability boundaries are avoided. Throughout the analysis the simplifying assumptions almost invariably are optimistic with regards to stability. For this reason it is considered desirable to obtain results of a simple analysis before a very detailed study is conducted.

The approximations, in addition to the standard ones (ref 10) are:

1) Forces, moments, and blade motions are quasi-static linear functions of the three aerodynamic variables $\Delta w, \Delta\dot{\theta}$, and Δq, and the two control inputs $\Delta\Theta_0$ and ΔB_{1s}. Implicit in this assumption is:

a) the neglect of unsteady aerodynamics, or aerodynamic lags, such as the familiar downwash lag, and

b) that rotor plane dynamics can be described by additional, redundant, equations in the helicopter variables.

2) There is considered to be a plane of longitudinal symmetry, and the motions in this plane are uncoupled from the lateral-directional motions.

3) The gusts are modeled as moving air mass fronts along body axes.

4) No attempt is made to simulate gust alleviation, gradual gust penetration, or gust spatial distribution. Gust alleviation, as it is used in this work, is defined in Appendix IV.

The equations are derived according to the procedure in Reference (11). The aerodynamic forces in the form of Taylor series, expanded in the three aerodynamic variables along the X, Z, and about the Y axis are summed with the control and inertial forces and moments in those directions and divided by the mass and the moment of inertia respectively. The terms are linearized for small perturbations and the sum set equal to the trim, or initial values. This leaves equations of the form:

$$- \Delta\dot{u} - X_u \Delta u - X_w \Delta w + \omega_o \Delta\dot{\theta} + g\Delta\theta = X_{B_1} \Delta B_1 + X_{\theta_o} \Delta\theta_o$$

$$(1) \quad - \Delta\dot{w} - Z_u \Delta u - Z_w \Delta w - U_o \Delta\dot{\theta} = Z_{B_1} \Delta B_1 + Z_{\theta_o} \Delta\theta_o$$

$$- M_u \Delta u - M_w \Delta w - M_{\dot{\theta}} \Delta\dot{\theta} = M_{B_1} \Delta B_1 + M_{\theta_o} \Delta\theta_o$$

The stability derivatives are dimensional derivatives with the units of 1/sec for the force derivatives and 1/sec-ft for the moment derivatives. They are defined as follows:

$$X_{(\,)} = \frac{1}{m} \frac{\partial X}{\partial (\,)} \qquad Z_{(\,)} = \frac{1}{m} \frac{\partial Z}{\partial (\,)} \qquad M_{(\,)} = \frac{1}{I_y} \frac{\partial M}{\partial (\,)}$$

where the partial derivatives indicate that four of the five variables are held constant while the derivative is taken respect to the fifth. This does not imply, however, that any dependent variable such as blade flapping, as it is treated in this work, is held constant during the process.

The equations describing the motion of the rotor plane (blade flapping) are, consistant with the assumptions made, independent redundant equations of the form:

$$a_1 = \frac{\partial a_1}{\partial w}\Delta w + \frac{\partial a_1}{\partial \dot\theta}\Delta\dot\theta + \frac{\partial a_1}{\partial u}\Delta u + \frac{\partial a_1}{\partial B_1}\Delta B_1 + \frac{\partial a_1}{\partial \theta_0}\Delta\theta_0$$

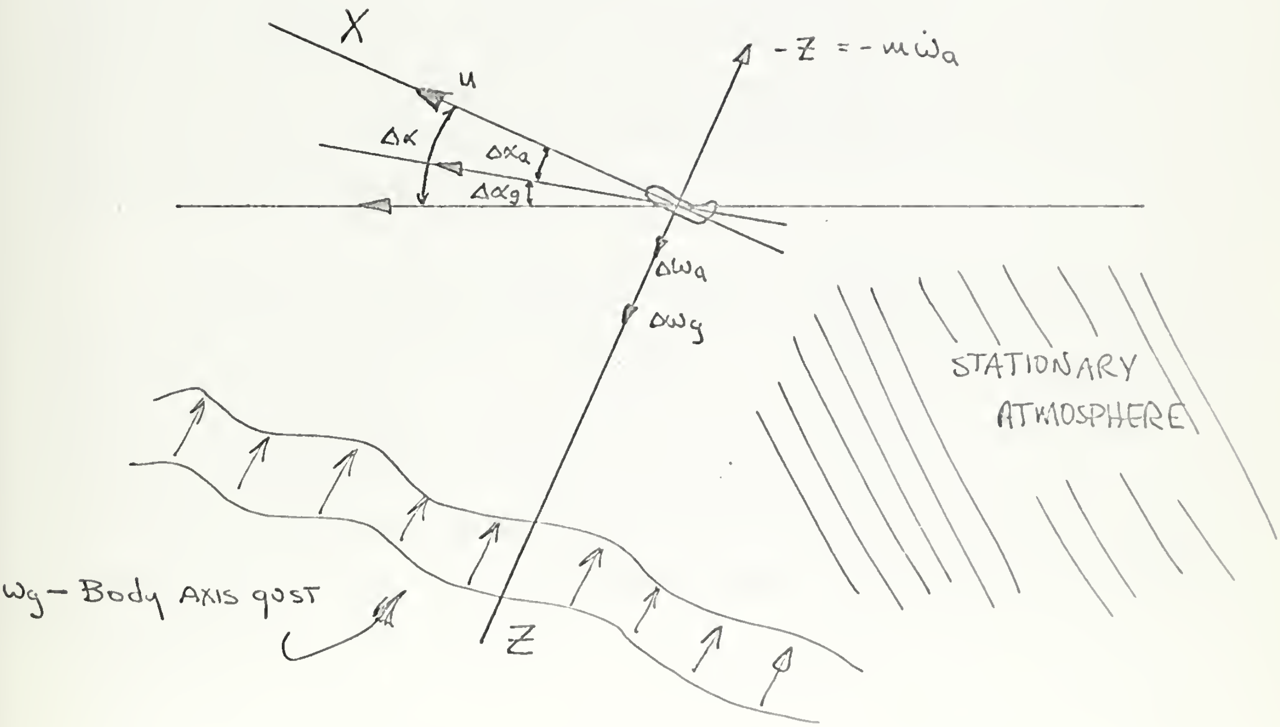
$$a_0 = \frac{\partial a_0}{\partial w}\Delta w + \frac{\partial a_0}{\partial \dot\theta}\Delta\dot\theta + \frac{\partial a_0}{\partial u}\Delta u + \frac{\partial a_0}{\partial B_1}\Delta B_1 + \frac{\partial a_0}{\partial \theta_0}\Delta\theta_0$$

This work pertains to the gust response of aircraft and so a suitable way of describing the gusts, especially for use on the analog computer and for ease of physical understanding, is used. The aerodynamic variable, Δw, is used to represent aircraft vertical velocity with respect to the local air mass such that

$$\frac{\Delta w}{U_0} = \Delta\alpha \qquad \text{angle of attack}$$

A gust, wg is modeled as the velocity of the local air mass with respect to the stationary atmosphere, and the aircraft velocity with respect to the stationary atmosphere is w_a. The sum is the aircraft velocity with respect to the local air mass such that $w = (\Delta wg + \Delta wa)$.

This substitution is made into equation (1), noting that the $\dot{w}$ term in the Z force equation is an inertial acceleration of a body, not an acceleration of the air mass, and should really be $\dot{w}_a$. All of this is shown in the figure which follows.



The equations can now be written in a form convenient for putting on the analog computer because the gusts are considered as separate inputs, or as forcing functions. They can be arranged as follows and patched into the computer in a standard manner.

$$-\Delta\dot{u}_a = \quad \cdot \quad \cdot \quad \cdot \quad X_w \Delta w_g$$

$$-\Delta\dot{w}_a = \quad \cdot \quad \cdot \quad \cdot \quad -Z_w \Delta w_g$$

$$-\Delta\ddot{\theta} = \quad \cdot \quad \cdot \quad \cdot \quad M_w \Delta w_g$$

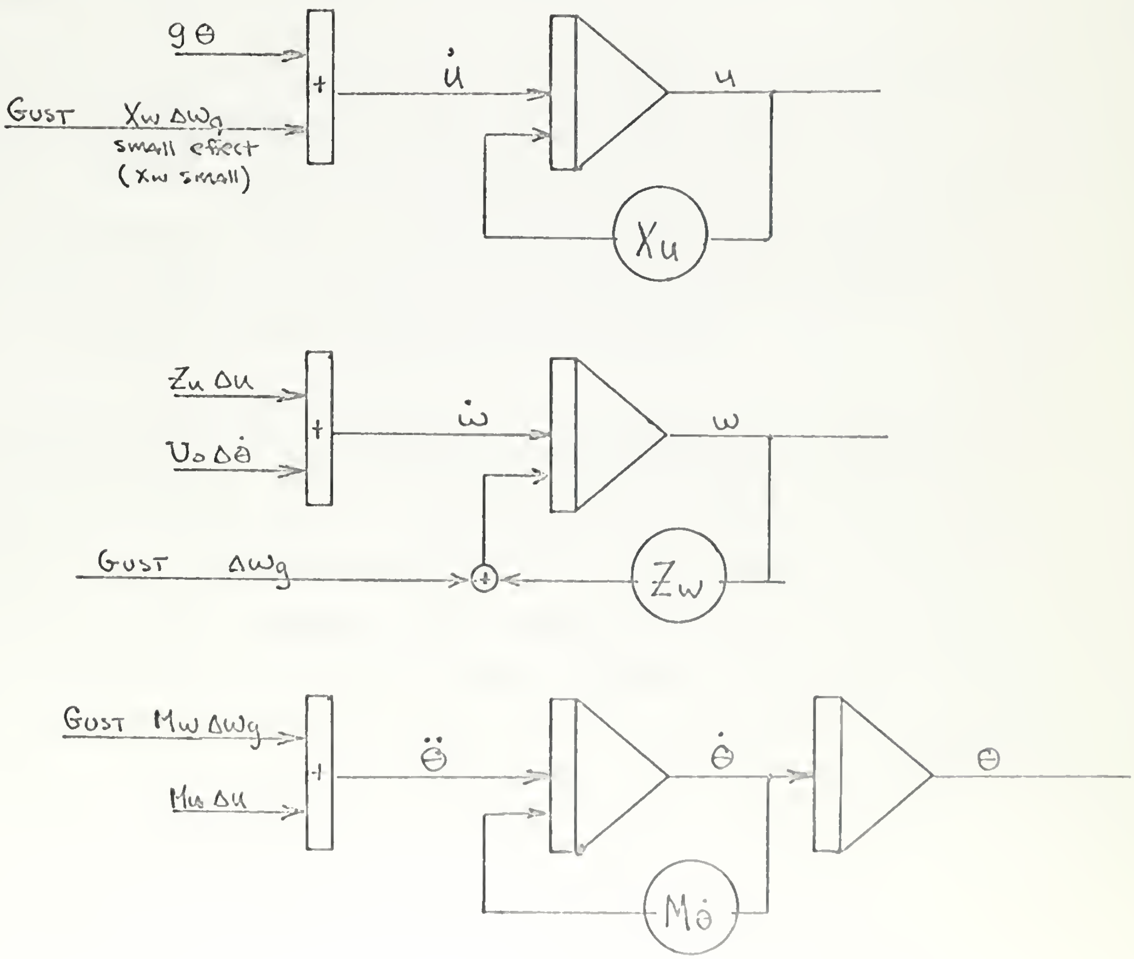
The left hand side represents the accelerations on the aircraft due to the right hand side. The analog computer diagram is presented as figure (1). The variables printed on the diagram are really "computer variables" because they are normalized to maximum values. The analog computer responses are labelled in dimensional units and all, except where noted, were made with a $7\frac{1}{2}$ ft/sec gust input. The values for all of the potentiometers were derived from the data from which the derivatives of Appendix I were taken.

It should be noted here that an analysis of this form differs from that where the transient response of the aircraft due to a control pulse input is studied. A control pulse input will cause different relative accelerations, along and about the three axes, from a gust disturbance, resulting in different aircraft motions. This can be seen as the difference between the numerators of the transfer functions for motions resulting from a collective input as opposed to a vertical gust input.

$$\frac{\Delta\theta}{\Delta\theta_o} = \frac{M_{\theta_o}(s+T_1)}{\Delta(s)} \qquad\qquad \frac{\Delta\theta}{\Delta(w_g)} = \frac{sM_w}{\Delta(s)}$$

The character of the motions are the same, but the actual responses are dependent on the numerators. When dealing with flying qualities the actual response is often of more importance than the character of the motion.

Much of this analysis will consist of the interpretation of analog computer runs, in contrast to the solution of transfer functions or mode ratios and mode shapes. The analog computer approach is taken in order to get a grasp on the basic nature of the responses and because of the ease in the analysis. As such it will be physically intuitive to keep in mind the simplified computer diagram which follows.



In this way one can see directly all of the parameters making up the airplane accelerations and, of importance in interpreting the modes of motion, can see the number of integrations in each loop. It is convenient to think in terms of electrical signals, adding and subtracting from one another and being integrated. In the second part of the thesis various feedbacks are investigated. Each is merely a combination of the helicopter variables summed together and called the feedback signal. This is shown in the full analog computer diagram of figure (1) where all of the different feedbacks are shown on the right side of the figure. Selection of one or more of the feedbacks is made by throwing the proper switch, which sums together a particular combination of all the variables and amplifies, attenuates, or integrates it according to the method chosen. The signal is then put into either the B_{1s} A/P or θ_0 A/P amplifiers on the left of the diagram as a control input.

Approximations to the equations of motion will be used frequently for ease of analysis. It will be shown in the first part that the approximations used are valid for the purposes of this work. This will be shown by comparing results of successively broader approximations, and, at the same time, will bring out the major differences between helicopter and airplane gust response characteristics.

It is necessary to have a standard of comparison for a study such as this. Unfortunately, there is no known single parameter that encompasses all of the important flying qualities parameters. Of even more concern is the lack of knowledge as to what the important flying qualities are. For the purposes here, however, rotational and normal acceleration magnitudes, as well as dynamic stability characteristics, are used as a general index of comparison. The acceleration magnitudes are important for structural and ride quality considerations, and the dynamic stability is important for control and flying qualities.

PART I

ANALYTICAL AND QUALITATIVE COMPARISON OF AIRPLANE AND HELICOPTER GUST RESPONSE CHARACTERISTICS.

In this part different approximations to the full three degrees of freedom of motion of an aircraft are considered. The value of each approximation is shown and the important points it reveals are discussed. As each approximation introduces stability derivatives into the equations they will be simply derived for both the helicopter and the airplane. Typical values for the derivatives are taken from data presented in the appendices, and in the references.

In order to gain an intuitive understanding of the various factors affecting an aircraft's response to a vertical gust, it is helpful to start as simply as possible. Consider an aircraft, or any body with some lift curve slope represented by the stability derivative Z_w, restrained from moving in any direction - bolted down in a wind tunnel for example. If the body is exposed to a vertical component of air velocity, w_g, then there is an aerodynamic force created on the body in the Z direction, and it could be measured by the wind tunnel balance. The force is

$$Z = \frac{\partial Z}{\partial w} \Delta w_g$$

Divided by the weight gives the normal load factor,

$$n_z = -\left(\frac{Z_w}{g}\right) \Delta w_g$$

If high frequency and, in the sense used here, higher order dynamics are neglected (e.g. blade dynamics, structural response), then this is the initial vertical gust response of an aerodynamic body. The higher order dynamic response is referred to as gust alleviation and effectively works so as to reduce the magnitude of the initial response. The load factor transfer function may be thought of as

$$\left(\frac{n_z}{w_g}\right)(s) = -\left(\frac{Z_w}{g}\right)\left(G(s)\right)$$

where G(s) is a dynamic transfer function which, for very low frequency inputs can be approximated by unity. (Appendix IV has a further discussion of gust alleviation.) This approximation then becomes a static approximation. It seems obvious that the derivative, Zw, will be important in the gust response characteristics of aircraft.

For a conventional airplane it can be shown (reference 11) that

$$Z_w = -\frac{g\,a_w}{U_0 C_L} \doteq \frac{U_0\,a_w}{2\mu\bar{c}} \qquad \text{airplane}$$

where $\mu = m/\rho S\bar{c}$ is airplane relative density and a_w is the slope of the lift curve for the whole airplane. Zw is proportional to airspeed but inversely proportional to size and relative density. Small, light, high aspect ratio airplanes flying at low altitude and high speed have large Zw's (order of-2) and heavy, high wing loading fighters flying slowly in approach have low Zw's (order of-0.2).

For the helicopter, Zw is markedly different. If all of the lift is considered to be from the thrust of the rotor,

$$Z_w = -\frac{1}{m}\frac{\partial T}{\partial w} = \frac{1}{m}\left\{\frac{\partial T}{\partial w_{ANF}} - \alpha_0 \frac{\partial T}{\partial u_{ANF}}\right\}$$

In terms of the rotor parameters Zw is proportional to

$$\frac{\partial C_T}{\partial \bar{w}} = \Omega R\frac{\partial C_T}{\partial w} = \frac{\left(\frac{a\sigma}{4}\right)\partial\lambda/\partial\bar{w}_{NF}}{\left[1 - \frac{a\sigma}{4}\,\partial\lambda/\partial C_T\right]} + \left(B_{1s} - \alpha_{f_0}\right)\frac{\frac{a\sigma}{2}\left(\theta_0\mu + \frac{1}{2}\,\partial\lambda/\partial\bar{u}_{NF}\right)}{\left[1 - \frac{a\sigma}{4}\,\partial\lambda/\partial C_T\right]}$$

The induced velocity parameters

$$\frac{\partial \lambda}{\partial \bar{\omega}_{ANF}}, \quad \frac{\partial \lambda}{\partial \bar{u}_{ANF}}, \quad \frac{\partial \lambda}{\partial C_T}$$

are taken from reference (12) where they are shown to approach unity, zero, and zero, respectively, as forward speed increases and induced effects become negligible. Using these high speed approximations and noting that, in level flight the second term will be negligible with respect to the first, then

$$\frac{\partial C_T}{\partial w} \doteq \frac{a\sigma}{4\Omega R} \qquad \text{for high speed.}$$

Dimensionalizing yields

$$Z_w \doteq -\frac{g\left(a/4\right)}{(\Omega R)\left(C_T/\sigma\right)} \qquad \text{helicopter (high speed)}$$

which is similar in form to that of the airplane, but is independent of airspeed. For a helicopter Zw approaches a constant value, for a given blade loading and RPM, at some high enough forward speed that induced effects are not important. It can be seen in the data of Appendix I that the speed range considered herein is not yet high enough to warrant this approximation because Zw for each helicopter is still increasing even at 180 kts. It is also interesting to note that the helicopter represented by the circle and the square are the same helicopter where the larger Zw corresponds to an empty weight condition and the lower Zw to a heavily loaded condition. The low "triangle" values are for another similar helicopter which is at its maximum gross weight.

In order to contrast airplane and helicopter Zw's it must be noted that Zw is highly dependent on the flight condition for a given airplane, whereas for the helicopter Zw approaches a constant with speed. The design flight condition thus enters the picture when a comparison is made. It is clear that at some speed and altitude (dynamic pressure) the helicopter and airplane will have the same Zw. Figure (2) indicates that, for six dissimilar aircraft for which data was readily available (ref. 10), the speed where this happens may be near the design condition for the particular airplane. The values of Zw for all six aircraft near their design flight condition were within the range of data for four similar helicopters at high forward speed. For purposes of comparison, it is thus considered that an airplane, designed to fly in the speed range of interest to this work, is likely to have a value of Zw similar to a helicopter flying at the same high speed. Typical values are on the order of unity. It is thus concluded that the static load factor for an airplane and a helicopter at the same high speed is similar.

The next approximation is a single degree of freedom approximation in which only vertical motion is allowed, the other two degrees (pitching and horizontal translation) being restrained. The body can be thought of as translating on a vertical rod thru its center of gravity. In this case

$$\Delta u = \Delta \theta = 0$$

so, of the equations of motion, the only one that applies is

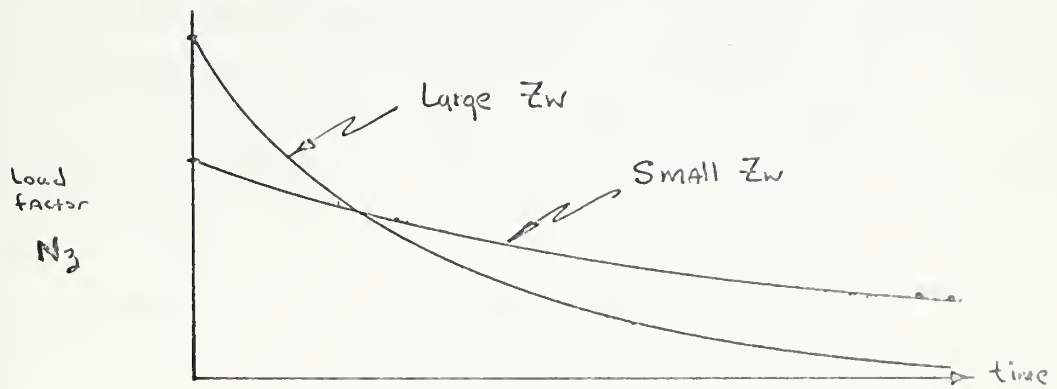
$$\dot{w}_a - Z_w \Delta w_a = Z_w \Delta w_g$$

The aircraft response to a steady vertical gust is

$$w_a = -w_g\left(1 - e^{-Z_w t}\right) \quad \text{"floating"}$$

$$N_z = -\frac{Z_w}{g}\,\Delta w_g\, e^{-Z_w t} \quad \text{normal acceleration}$$

This simple approximation shows that the time constant of normal acceleration is also dependent on Zw. In the figure are shown typical responses for different Zw's.



The large Zw causes a higher peak load factor, but a faster time constant in reducing the load to zero. Statically, then, a small Zw is desirable, but dynamically the larger Zw is seen to be better because of the increased rapidity with which the gust induced accelerations are reduced to zero.

The approximation obviates the fact that a lifting surface, airplane, or helicopter will "float" up in response to a vertical gust until the aircraft's vertical velocity is equal to that of the gust and the angle of attack is thus zero. The longer it takes for this to happen, the longer will the aircraft experience a normal acceleration, as well as other forces and moments due to a non-zero angle of attack, (e.g. $M_\alpha \Delta\alpha$, $X_\alpha \Delta\alpha$).

From the first two simple approximations it is seen that Zw, the lift curve slope of the aircraft, controls the load factor, and that automatic direct lift or thrust control could be used to either decrease the static load or decrease the time constant of the response, but could not accomplish both objectives at the same time.

The next step is to consider two degrees of freedom, pitching and heaving, at constant forward speed. This is the so called short period approximation of the conventional airplane, but really is an approximation to the motions ensuing before airspeed has a chance to change appreciably. The approximation is exact for the time it takes for three integrations, if Xw = 0. (This can be seen on the simplified analog diagram in the Description of Analysis Section by tracing a signal entering as w_g and finally causing a Δu_a).

The equations that govern these motions are

$$\Delta \dot{w}_a - Z_w \Delta w_a - U_0 \Delta\dot{\theta} = Z_w \Delta w_g$$

$$\Delta \ddot{\theta} - M_{\dot{\theta}} \Delta\dot{\theta} - M_w \Delta w_a = M_w \Delta w_g$$

and the characteristic equation is

$$(s - Z_w)(s - M_{\dot{\theta}}) - M_\alpha = 0$$

It was found that Zw is likely to be of the same magnitude for the helicopter and the airplane, and now $M_{\dot{\theta}}$ and M_{α} must be investigated for contrast. The airplane will be considered first.

For an airplane the moment derivatives depend heavily on the horizontal tail. The only contribution to $M_{\dot{\theta}}$ usually accounted for in estimates for "conventional" airplanes comes from the change in tail lift with tail local angle of attack as the airplane pitches. One way of writing the expression for $M_{\dot{\theta}}$ is:

$$M_{\dot{\theta}} = \frac{l_t}{I_y} \frac{\partial L_t}{\partial \alpha_t} \frac{\partial \alpha_t}{\partial \dot{\theta}} = l_t^2 S_t \frac{\rho U_{o\,(tail)}}{2 I_y} \left.\frac{\partial C_L}{\partial \alpha}\right|_{tail}$$

$$M_{\dot{\theta}} = \frac{l_t U_o}{2 K_y^2 \mu} C_{m_{i_t}} \qquad \text{airplane}$$

where $U_o = U_o(\text{tail})$ (i.e. the absence of power effects)

and $V_t = \frac{S_t l_t}{S \bar{c}}$, $\qquad C_{m_{i_t}} = a_t V_t$

The magnitude of $M_{\dot{\theta}}$ for the airplane thus depends on the tail size and the square of the tail length, and is proportional to forward speed. Typical values in the speed range of interest are between -1.0 and -2.0, as taken from the data in reference (10).

An expression for M_α is easily written from a force diagram by summing all the moments about the center of gravity

$$\Sigma M = 0 = (x_{CG} - x_{ac}) L_w + M_f - l_t L_t$$

and taking the derivative

$$\frac{\partial M}{\partial \alpha} = (x_{CG} - x_{ac}) q S a_w + q S \bar{c} C_{m_{\alpha_f}} - q_{(tail)} S_t l_t a_t$$

In the absence of power effects q = q (tail) and

$$M_\alpha = \frac{U_o^2}{2 K_y^2 \mu} \left[\frac{(x_{CG} - x_{ac})}{\bar{c}} a_w + C_{m_{\alpha_f}} - C_{m_{i_t}} \right]$$

For the "conventional" airplane, M_α is thus proportional to dynamic pressure and depends on center of gravity position, tail length, tail size and fuselage characteristics. The tail term is usually the dominant one in the expression such that M_α is large and negative to provide ample static stability.

The "conventional" helicopter is not at all like the airplane in regards to M_α and $M_{\dot{\theta}}$. For the conventional single rotor helicopter with no hinge offset or horizontal tail, and neglecting the fuselage, $M_{\dot{\theta}}$ arises from the lag in the tilt of the resultant thrust vector as the shaft pitches. For these helicopters the lag of the thrust vector is approximately equal to the lag of the tip path plane of the rotor (reference 1). M_α arises from the increased drag on the rotor blades (H force) as angle of attack is increased. Both of the derivatives depend on hub height, and are independent of center of gravity position. For the hub above the center of gravity $M_{\dot{\theta}}$ is negative (stable damping term), and M_α is positive (static instability).

The helicopters capable of flying in the 150 kts range, and especially the large, heavy ones of this study, are very unlike the "conventional" helicopter. These newer helicopters have main rotor flapping hinge offsets, horizontal tails, and large, bulky fuselages. The reason for the hinge offset is for greater control power, and the tail becomes necessary (and effective) as speed increases. This work is concerned with only these latter types of helicopters and all reference will be to the helicopters represented by the data in Appendix I.

From the force balance diagram in Appendix (II) comes

I (1)
$$\Sigma M = 0 = T\left[\bar{h} - h(B_{1s} + i_s)\right] + H\left[h + \bar{h}(B_{1s} + i_s)\right] + \frac{e b M_s \Omega^2}{2} a_{1s} + M_f - l_t L_t$$

When the small angle approximation, $H \doteq T a'$ is made, the derivative taken with respect to $\dot{\theta}$, and it is noted that

$$1 \gg \frac{\bar{h}}{h}(B_{1s} + i_s) \, ,$$

then

$$\frac{\partial M}{\partial \dot{\theta}} = h T \frac{\partial a'}{\partial \dot{\theta}} + \frac{e b M_s \Omega^2}{2} \frac{\partial a_1}{\partial \dot{\theta}} - \frac{l_t}{U_0} q_t S_t a_t$$

From the relation $a' = C_H / C_T$, it can be shown that (reference 1)

$$\frac{\partial a'}{\partial \dot{\theta}} = \frac{4}{3} \frac{a}{\gamma \Omega} \left[\frac{18}{a} - \frac{\theta_0}{C_T/\sigma}\right] - h \frac{\partial a'}{\partial u}$$

As $\theta_o / \left(\frac{C_T}{\sigma}\right)$ increases, as with heavier faster helicopters with the rotor plane tilted farther forward, it is seen that $\partial a_1' / \partial \dot{\theta}$ decreases.

It has been found characteristic of the helicopters investigated that this term results in a small contribution to the pitch damping at 150 kts. The trend of $\partial a_1' / \partial \dot{\theta}$ with speed for the data helicopters is shown in figure (3). For the 24000 lb helicopter, for instance, this term (which makes up all of the pitch damping in the "conventional" helicopter) contributes less than five percent to the total damping.
From this information it is seen to be reasonable to neglect the tilt of the rotor resultant force vector in estimating the pitch damping at high speeds.

The time constant of the rotor tip path plane tilting response to a shaft tilt rate is

$$\frac{\partial a_1}{\partial \dot{\theta}} = \frac{16}{\gamma \Omega \left(1 - \frac{\mu^2}{2}\right)} + h \frac{\partial a_1}{\partial u}$$

which, for the speed range and helicopters of interest can be approximated by $16/\gamma\Omega$. Including these assumptions results in

$$M_{\dot{\theta}} = -\frac{8 e b M_s \Omega^2}{\gamma I_y} - \frac{(\rho/2)\, U_o\, l_t^2\, S_t\, a_t}{I_y} \eta$$

where $\eta = q_{tail}/q_o$ the ratio of the dynamic pressures in and out of the rotor slipstream at the tail.

For the data helicopters the table below shows the approximate breakdown for the contribution to $M_{\dot\theta}$ at 150 kts.

HELICOPTER	$\dfrac{\theta_0}{(C_T/\sigma)}$	$M_{\dot\theta}$	TAIL TERM	HINGE OFFSET TERM
24000 fwd	3.1	.77	75%	20%
35000 aft	3.0	.54	60%	35%
42000 fwd	2.76	.38	40%	40%
18000 aft	2.97	.53	50%	35%

Typical values for the pitch damping are seen to be between -0.5 and -1.0 at 150 kts. It is clear that the need for a tail increases with $\theta_0 / (\frac{C_T}{\sigma})$ (decreasing $\partial a' / \partial \dot\theta$), and the data is in agreement with the prediction of reference (1) which says that the tilt of the thrust vector contribution to the pitch damping should be zero for

$$\theta_0 / (\frac{C_T}{\sigma}) \doteq 3.0$$

As tail size increases the pitch damping approaches that of the airplane with the same tail size.

An expression for M_α results from taking the derivative of equation I (1) with respect to α.

$$\frac{\partial M}{\partial \alpha} = T \frac{\partial a'}{\partial \alpha}\left[h + \bar{h}(B_{1s} + i_s)\right] + \frac{\partial T}{\partial \alpha}\left\{a'\left[h + \bar{h}(B_{1s}+i_s)\right]\right.$$

$$\left. + \bar{h} + h(B_{1s}+i_s)\right\} + \frac{e b M_s \Omega^2}{2}\frac{\partial a_1}{\partial \alpha} + \left.\frac{\partial M}{\partial \alpha}\right|_{tail} + \left.\frac{\partial M}{\partial \alpha}\right|_{fuselage}$$

Making the approximation that, for any helicopter $h \gg \bar{h}(B_{1s}+i_s)$,

using the high speed, approximate expression for $\partial C_T / \partial \omega$ derived for Z_w earlier, and making the approximation that, in the absence of induced effects

$$\frac{\partial a_1}{\partial \alpha} \doteq \frac{2\mu}{\left(1-\frac{\mu^2}{2}\right)} \frac{\partial \lambda}{\partial \alpha} \doteq 2\mu^2$$

then, the expression for M_α becomes

I (2)
$$M_\alpha = \frac{g h}{K_y^2}\left[\frac{\partial a'}{\partial \alpha} + \frac{a U_0 \epsilon}{4(\Omega R)(C_T/\sigma)}\right] + \frac{e b M_s \Omega^2 \mu^2}{I_y}$$

$$+ q_0 \frac{\pi R^3}{I_y}\left[\eta C_{m_{i_t}} + C_{m_{\alpha_f}}\right]$$

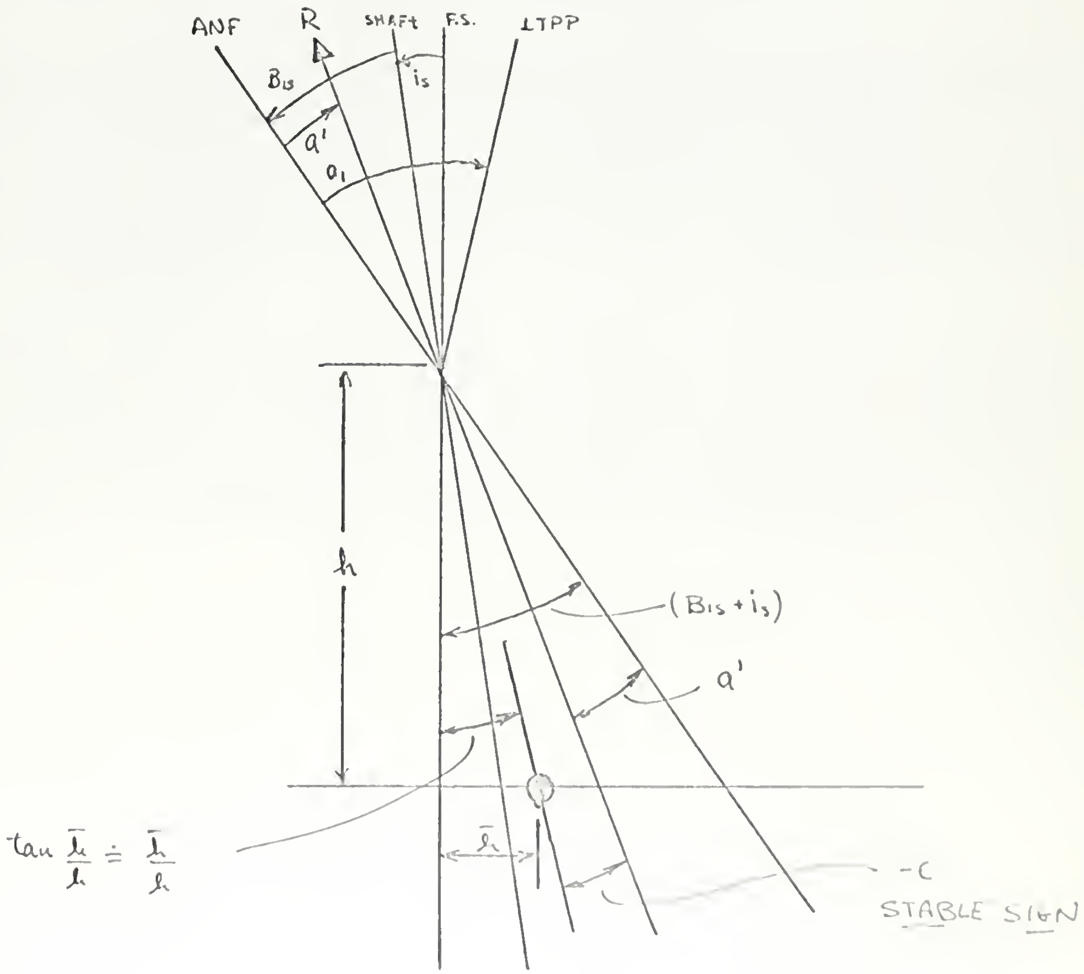
where

$$C_{m_{i_t}} = -a_t V_t = \frac{a_t S_t l_t}{\pi R^3} \quad , \quad M_f = q_0 \pi R^3 C_{m_f}$$

$$\epsilon = \left[a' + \frac{\bar{h}}{h} - (B_{1s}+i_s)\right]$$

All of the terms except $C_{m_{i_t}}$ and, possibly, the second one are destabilizing. As in the airplane, the tail should be large enough to overcome the instability of the rest of the aircraft. Unlike the airplane, however, there are other restrictions, such as control to trim at different power settings, that govern the size of the tail and have kept it relatively small on all present helicopters (relative to the airplane of the same gross weight). It can be seen from the expression above that there is little chance for M_α to be negative without an extremely large tail.

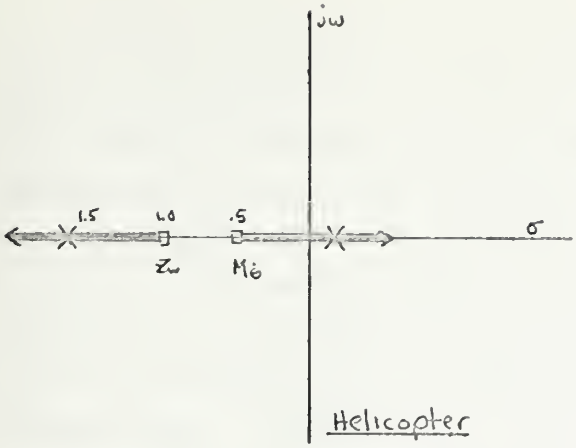
It should also be remembered that the fuselage is characteristically large and bulky for these helicopters, resulting in a much larger fuselage contribution to the static instability as forward speed increases than for the airplane of corresponding gross weight. The argument, from a stability point of view, is for a much larger tail, but as said, other factors are likely to keep the tail size smaller than on the corresponding airplane. The tilt of the resultant thrust vector term and the hinge offset term are both destabilizing, and both increase with a power of the forward velocity as can be seen from figure (4). The second term in the expression can be stabilizing or destabilizing, depending on the orientation of the resultant rotor force vector, R, with respect to the center of gravity in the trimmed flight condition of interest. The term is stabilizing if the extension of the vector, R, is behind the center of gravity, ϵ is negative, as in the figure.

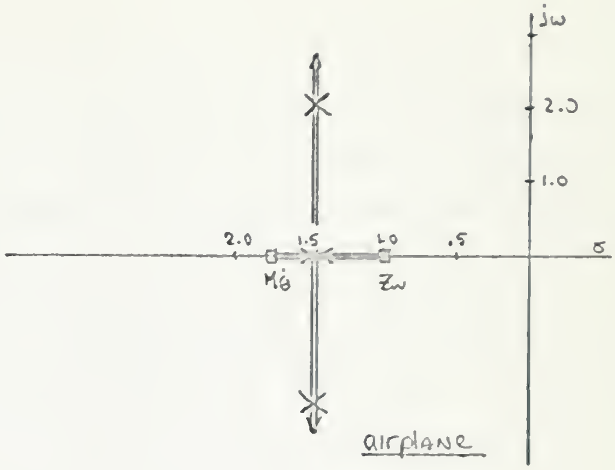


In order for this to be the case, the net moment in the trimmed flight condition contributed by the flapping of the blades about the offset hinges, the fuselage, and the tail must be positive. This means that the tail should be at negative incidence and thus carry a download to increase the size of this stabilizing term. A download is also desirable to increase static speed stability, Mu, but other factors such as dynamic stability, and control required versus trim speed and power determine the incidence of the tail. It should be noted that, for the "conventional" helicopter, neglecting the fuselage, this term is identically zero because the force vector, R, must pass thru the center of gravity in trim. For helicopters with hinge offset, tails, and large fuselages, however, this term is, in general, non-zero and increases with forward speed. It is also seen that the center of gravity position plays a smaller role in the static stability of the helicopter than for the airplane. The C.G. position, $\bar{h}/l$, only occurs in the ϵ term and its importance is obscured and diminished as all the other terms in equation I (2) get more important.

The actual value of M_α is very dependent on the particular configuration as each term in the expression can be of the same order of magnitude. It is probable, however, that M_α will be positive or unstable for present helicopters in the high speed range. The average value for the data helicopters at 150 kts is about +2.0.

Returning now to the characteristic equation for the constant speed, $\Delta u = 0$, approximation, if "typical" poles are plotted on the complex plane, and the roots traced as M_α is varied to "typical" values for the airplane and helicopter, the differences in characteristics become apparent.





For the helicopter, instability occurs when

$$\left(M_\alpha - Z_w M_{\dot\theta}\right) > 0$$

which, for "typical" values of Zw and $M_{\dot\theta}$ given earlier, is when M_α is between +0.5 and +1.0. This is below the values for all but one of the data helicopters, and the effect can be clearly seen in the analog computer responses for the different helicopters in figure (5). In all but the most stable one the motions are dominated by a rapid divergence in pitching - a consequence of angular acceleration induced by positive M_α.

For the airplane, the roots are of short period and well damped, lying in the region indicated on the figure. This means that the airplane will accelerate to the gust velocity very quickly, and the moments and forces due to angle of attack will be quickly dissipated after a disturbance by a steady vertical gust.

A further approximation to the equations of motion to adequately describe airplane accelerations in response to steady gusts is not needed. The airplane typically exhibits a phugoid, or long period motion, in Δu and $\Delta\theta$ while $\Delta w = 0$ (in the body axis system). The resulting accelerations on the airplane are small compared to the ones in the "short period" response. For the helicopter, however, the pitching and angle of attack responses are influenced by the changes in airspeed because of the nature of the derivative M_u, the forward speed static stability. For the "conventional" airplane, Mu = 0, but for the conventional, as well as the high speed large helicopter, Mu is in general non-zero. An expression for Mu can be written as

$$\frac{\partial M}{\partial u} = hT\frac{\partial a'}{\partial u} + \frac{ebM_s\Omega^2}{2}\frac{\partial a_1}{\partial u}$$

$$+ \pi R^3 q_0 \left\{ \left(\frac{\partial C_m}{\partial u} + \frac{2C_m}{U_0} \right)_{fuselage} - \eta \bar{V}_t \left(\frac{\partial C_L}{\partial u} + \frac{2C_L}{U_0} \right)_{tail} \right\}$$

where ($\frac{\partial C_m}{\partial u}$) (fuselage) and ($\frac{\partial C_L}{\partial u}$) (tail) are influenced by induced effects or changes of the rotor slipstream with speed. It can be seen that the size and magnitude may depend largely on the incidence of the horizontal tail, if the tail is large. Positive Mu is statically stable, but, as will be shown later, positive Mu leads to dynamic instability. All of the terms in the expression vary somewhat irregularily with forward speed because of the induced effects of the slipstream, even at speeds as high as 150 kts for the data helicopters. With a large enough tail, at high speed, the sign of the term can be assured by adjusting the tail incidence, but, with a small tail, the value of Mu is very dependent on the particular configuration and flight condition (e.g. level or non-level flight). Of the data helicopters, the two that exhibited the highest and lowest values of Mu both carried tail down loads, while the two that exhibited little change with speed carried a small positive or zero load at 150 kts. The smallest Mu's were for the smallest helicopter with the smallest tail.

An approximation that includes only this particular effect of the airspeed changes is presented as follows:

It is assumed that Xu, Xw << g and Zu = 0.

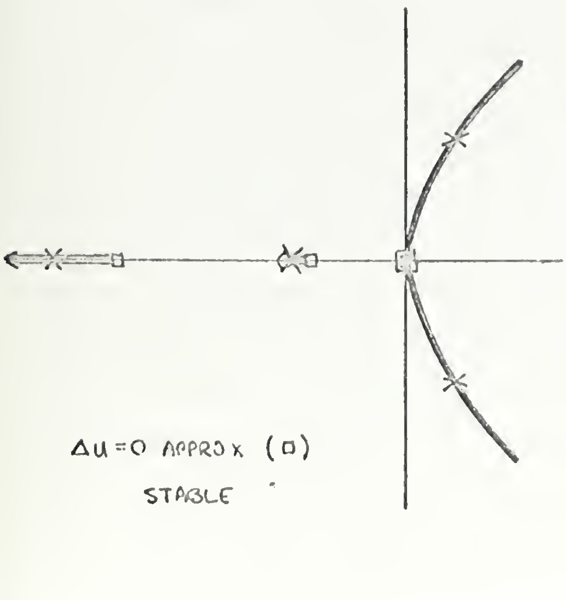
The characteristic determinant can then be written as

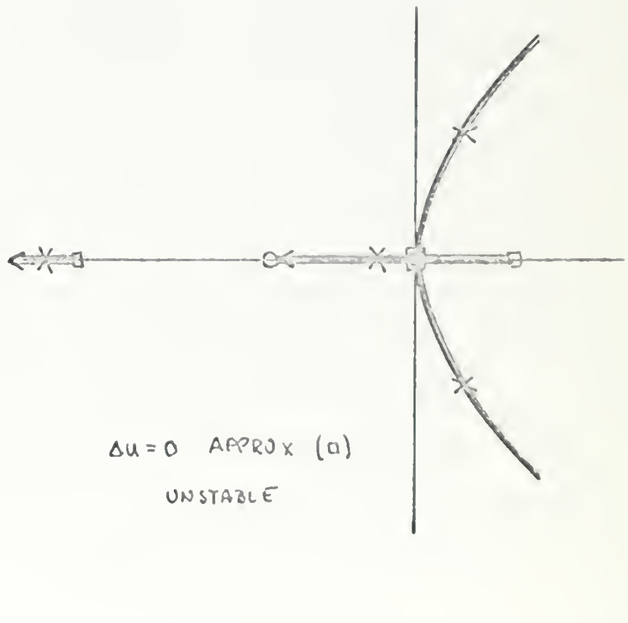
$$\begin{vmatrix} s & 0 & g \\ 0 & s-Z_w & -U_o s \\ -M_u & -M_w & s(s-M_{\dot{\theta}}) \end{vmatrix} = 0$$

and the characteristic equation, in root locus form,

$$s^2\left(\Delta u = 0 \text{ APPROX ROOTS}\right) + gM_u\left(s - Z_w\right) = 0$$

For a positive Mu, which is taken from the data as typical for these kinds of helicopters, the root loci for increasing Mu will look approximately like one of the following





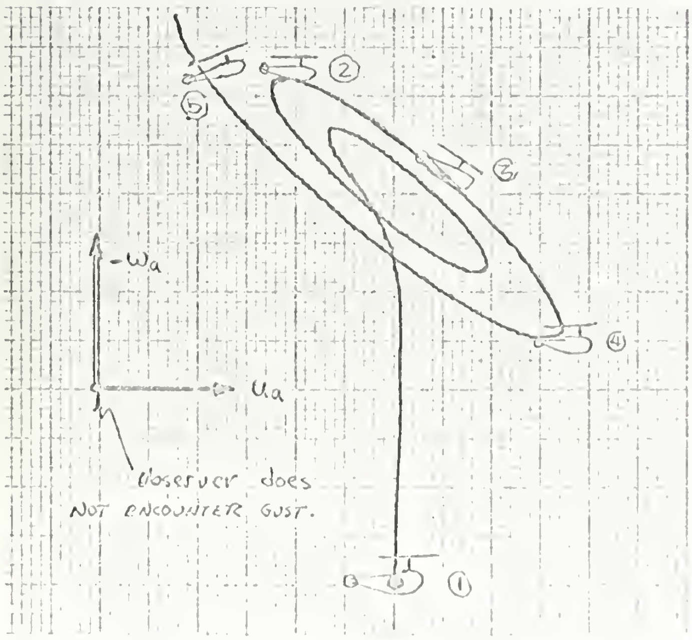
From the figure it can be seen that, regardless of the $\Delta u = 0$ root locations, the helicopter will always exhibit two real convergences if Mu is positive. These roots can loosely be thought of as the convergences of angle of attack and pitching rate, the larger one being the faster angle of attack response. The inclusion of the other Δu related derivatives is shown in the analog computer responses of figure (6), where

a) is for a helicopter with the "typical" derivatives (except for Zw and Mw)
b) is for the lowest values of all the derivatives (except Zw and Mw) and
c) for their highest values (except for Zw and Mw).

It is seen that there is very little difference in the acceleration responses, N_z and $\ddot{\theta}$, for the first few ($\doteq 10$) seconds throughout the range of all these derivatives. The reason for this is that the rate of convergence in angle of attack depends primarily on Zw, and the accelerations are primarily dependent on the angle of attack. For the first few seconds the acceleration response is almost independent of anything except Mw and Zw and, as such, should be accurately approximated by the two degree of freedom model in which $\Delta u = 0$. This corresponds to the fast convergent root which represents the $\Delta\alpha$ motions before coupling with Δu motions.

The result of all of the discussion above is the point that there should be little difference in the first few seconds of airplane and helicopter normal acceleration responses, regardless of the stability or instability of the two degree or three degree of freedom models. The static and the one degree of freedom models show, with the assumption that Zw is of the same magnitude, that the responses will be identical. Inclusion of pitching as a degree of freedom only changes the time constant of the normal acceleration convergence, making it longer with positive M_α and shorter with negative M_α. If M_α is large and positive the convergence becomes a divergence and inclusion of airspeed as a degree of freedom is necessary to describe the motion, but when this is done (for positive Mu) it is seen that a convergence reappears on the complex plane. Physical interpretation of this will be given in the following qualitative discussion.

Regardless of root locations or characteristic modes of motion, there are some fundamental differences between airplane and helicopter gust responses that do not show up on the complex plane. The biggest, most obvious, and most fundamental is the fact that with different signs of M_α, helicopters and airplanes pitch in different directions in response to a vertical gust. If M_α for an aircraft were zero, its initial response to a vertical gust would be pure heave - no pitching at all - and a very slow and small response in Δu whose sign depends on Xw. (Xw can be positive or negative but is always small.) For the airplane with negative M_α the response is a nose down pitching along with the vertical acceleration. This causes an increase in airspeed as the important Δu response (for $Xw \ll g$) due to the $g\theta$ term in the X force equation. The airplane then enters into its characteristic long period, or phugoid, oscillation. The opposite initial response occurs for the helicopter with positive M_α. The helicopter pitches up, and the important Δu response is a loss of airspeed. In addition, if Mu is positive, as it is with all of the data helicopters, the Δu response decreases the pitching acceleration and tends to cause an oscillatory mode if the effect is strong enough. The following diagram illustrates the differences between the two aircraft by showing what an observer, flying straight and level at a constant airspeed, would see when the two aircraft encountered gusts.



When the helicopter oscillatory mode is of a long enough period, the major part of the acceleration responses will be convergent, like the airplane, and will take place in the first few seconds. A pilot would probably not notice the differences in the normal acceleration response for the two aircraft. In figure (7) are shown analog computer responses for the same aircraft but with

a) positive M_α,

b) $M_\alpha = 0$, and

c) negative M_α.

Both the constant speed approximation and the three degrees of freedom responses are shown, and they illustrate the observations made above.

As a conclusion to this part of the work, it is possible to summarize the findings qualitatively as the differences a pilot would encounter in the two types of aircraft as a result of a gust disturbance. First, he would probably not notice the difference in normal acceleration, either the maximum load or the first few seconds of the dynamic response. Second, and most important, he would notice the different direction of rotational acceleration, especially if he is far ahead of the center of gravity. In the helicopter he will experience a local "g" response greater than that of the airplane because the helicopter pitches nose up. Third, he will notice a smaller angular damping and a tendency for the helicopter to continue to rotate until the static speed stability causes restoring moments, usually resulting in a long period unstable oscillation. Finally, because the pilot is able to respond with his controls, he will notice the difference in control response required of him in order to dissipate the accelerations. In the airplane the pilot pulls his stick back to keep his nose up in response to a vertical gust, and in the helicopter he must push forward on the cyclic control to keep his nose down.

PART II

ROTOR FLAPPING FEEDBACK CONTROL OF GUST INDUCED HELICOPTER DYNAMICS

This part of the thesis pertains to the elementary stabilization of the heavy, high speed, single rotor helicopter which was characterized in the first part of the thesis. The stabilization schemes that are investigated include a conventional rate and attitude system which is compared and contrasted with various types of rotor flapping, tilting moment, and thrust feedbacks. The impetus is to determine whether or not the latter are adequate to provide desirable gust response qualities for a helicopter.

It is often considered desirable to reduce the amount of rotor flapping that occurs due to a gust disturbance. This is essentially an attempt at devising an "inherently stable" rotor with which to stabilize the helicopter. The reduced flapping is generally favorable to blade motion stability, but the extent to which this "improves" the dynamics of the helicopter is not given proper attention. Delta Three and Oehmichen pitch flap coupling, and variations of these, the Lockheed gyro system, and a hub moment sensing system are examples of these techniques. In most of the literature, however, only the "fixed shaft" dynamics of the rotor, equipped with the particular device, is studied. The consequence of this is that the actual effect on a helicopter's flying qualities is not known. In this part of the thesis a simplified investigation of the effects on the helicopter as would be encountered in actual flight are investigated.

Influence of Rotor Flapping on Stability Derivatives

The fundamental problem in trying to determine how rotor flapping influences helicopter stability is to determine how much of the total forces and moments on the helicopter is attributable to flapping. An assumption often made with the low speed conventional helicopter is that the rotor resultant force vector, R, is perpendicular to the rotor tip path plane, and that the motions of the two coincide after a disturbance. This assumption leads to the erroneous conclusion that the position, or rate of change of position, of the rotor tip path plane due to a disturbance or control input is more important in the stability and control of the helicopter than it really is. Stability only is considered in this work, but the same arguments as will be presented can be used for synthesis of helicopter control systems.

Precisely speaking, helicopter accelerations only partially depend on rotor flapping, and as power, speed, and size increase, these other factors become of prime importance. Because of the way in which rotor flapping is treated in this work, i. e. as a dependent variable whose magnitude depends only on the helicopter variables, the influence of flapping on the stability derivatives is implicit. In order to expose the flapping dependent contributions to the derivatives, it is necessary to consider the flapping angles as "quasi-independent" variables and to take partial derivatives holding them constant. A moment derivative will thus look like

$$\frac{\partial M}{\partial (\,)} = \left.\frac{\partial M}{\partial (\,)}\right|_{a_1 a_0 b_1} + \left.\frac{\partial M}{\partial a_1}\right|_{a_0 b_1} \frac{\partial a_1}{\partial (\,)} + \left.\frac{\partial M}{\partial b_1}\right|_{a_0 a_1} \frac{\partial b_1}{\partial (\,)} + \left.\frac{\partial M}{\partial a_0}\right|_{a_1 b_1} \frac{\partial a_0}{\partial (\,)}$$

Expanding, because $a' = a'(a_1, a_0, b_1)$ results in

$$\frac{\partial M}{\partial (\,)} = \left.\frac{\partial M}{\partial (\,)}\right|_{a_1 b_1 a_0} + \frac{\partial a_1}{\partial (\,)}\left[\left.\frac{\partial M}{\partial a'}\right|_{a_1}\frac{\partial a'}{\partial a_1} + \left.\frac{\partial M}{\partial a_1}\right|_{a'}\right]_{b_1 a_0}$$

II (1)
$$+ \frac{\partial b_1}{\partial (\,)}\left[\left.\frac{\partial M}{\partial a'}\right|_{b_1}\frac{\partial a'}{\partial b_1} + \left.\frac{\partial M}{\partial b_1}\right|_{a'}\right]_{a_0 a_1}$$

$$+ \frac{\partial a_0}{\partial (\,)}\left[\left.\frac{\partial M}{\partial a'}\right|_{b_1}\frac{\partial a'}{\partial a_0} + \left.\frac{\partial M}{\partial a_0}\right|_{a'}\right]_{b_1 a_1}$$

Each term in the equation has a special significance, which is listed below.

$\left.\frac{\partial M}{\partial (\,)}\right|_{a_1 b_1 a_0}$ Change of moment with ( ) independent of rotor flapping. Fuselage, tail, tilt of the R vector independently of the TPP, and thrust changes make up this term.

$\frac{\partial M}{\partial a'}\frac{\partial a'}{\partial a_1}$ That component of the tilt of the R vector (H force) dependent on the longitudinal tilt of the TPP.

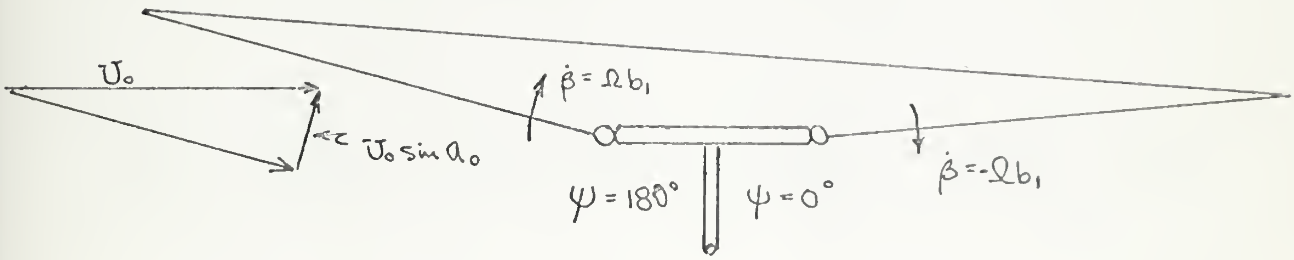
$\frac{\partial M}{\partial a'}\frac{\partial a'}{\partial a_0}$, $\frac{\partial M}{\partial a'}\frac{\partial a'}{\partial b_1}$ That part of the tilt of the R vector (H force) dependent on coning and lateral flapping angles.

$\left.\frac{\partial M}{\partial a_1}\right|_{a'}$ Direct hub moment due to hinge offset (or flapping restraint, or any other direct cause of moment due to the mechanical aspect of flapping as opposed to the aerodynamic forces caused by flapping.)

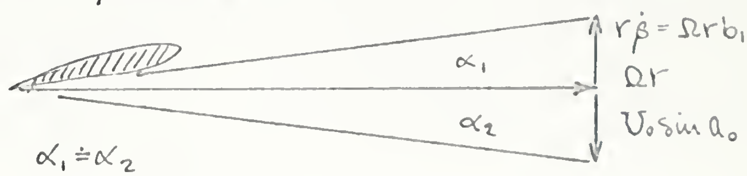
$\left.\frac{\partial M}{\partial b_1}\right|_{a'}$, $\left.\frac{\partial M}{\partial a_0}\right|_{a'}$ Zero. (all helicopter variables are held constant while the partial derivative is taken.)

The term, $\left(\frac{\partial a'}{\partial a_1}\right)$ can be thought of as a measure of how closely the rotor resultant force, R, and the perpendicular to the tip path plane, TPP, coincide. It is like a correlation function in that when the derivative is unity the R vector is perpendicular to the TPP and when the derivative is zero the orientation of the R vector is independent of the TPP. The terms $\partial a'/\partial a_0$ and $\partial a'/\partial b_1$ can be thought of as dihedral effect due to the coning and longitudinal blade flapping velocities, $\dot{\beta}$.

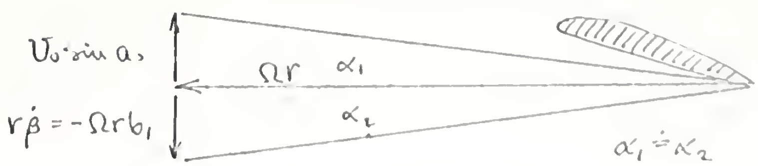
The following figure illustrates this.



$\psi = 180°$ BLADE ELEMENT



$\psi = 0°$ BLADE ELEMENT



It is seen that the longitudinal flapping velocity (lateral tilt, b_1) reduces the dihedral coning angle effect, thus decreasing the backward tilt of the R vector. It will be shown analytically in the coming discussion that the two effects almost cancel.

The important distinction to be made in this analysis is the partial independence of a' and a_1, and the important result is "by how much?". The angle a' can be written as

$$a' = C_H / C_T$$

where
$$\frac{2C_T}{a\sigma} = \frac{\theta_0}{3}\left[1 + \frac{3}{2}\mu^2\right] + \frac{\lambda}{2}$$

$$\frac{2C_H}{a\sigma} = \left[\frac{\delta\mu}{2a} + \left(\frac{\theta_0}{3} + \frac{3\lambda}{4}\right)a_1 - \frac{\mu\lambda\theta_0}{2} + \frac{\mu a_1^2}{4} - \frac{a_0 b_1}{6} + \frac{\mu a_0^2}{4} - \frac{16}{\gamma\Omega}\Delta\dot{\theta}_f\left\{\frac{2C_T}{a\sigma}\left(1 - \frac{7}{6}\mu^2\right) + \frac{\lambda}{4}\left(1 - \frac{\mu^2}{2}\right)\right\}\right]$$

$$a_0 = \frac{\gamma}{8}\left[\theta_0\left(1 + \mu^2\right) + \frac{4}{3}\lambda\right]$$

$$a_1 = \frac{1}{\left[1 - \frac{1}{2}\mu^2\right]}\left[\mu\left(\frac{8}{3}\theta_0 + 2\lambda\right) - \frac{16}{\gamma\Omega}\Delta\dot{\theta}_f\right]$$

$$b_1 = \frac{1}{[1+\frac{1}{2}\mu^2]}\left[\frac{4}{3}\mu a_0 - \frac{\Delta\dot{\theta}_f}{\Omega}\right]$$

From the expressions above can be found the derivatives

$$\frac{\partial a'}{\partial a_0}\ , \qquad \frac{\partial a'}{\partial a_1}\ , \qquad \frac{\partial a'}{\partial b_1}$$

They are, after much algebra:

(a) $$\left.\frac{\partial a'}{\partial a_0}\right|_{a_1 b_1} = \frac{a\sigma}{2C_T}\left[\frac{\mu a_0}{2C_T}\left(1 - \frac{4}{9}\,\frac{1}{[1+\frac{1}{2}\mu^2]}\right) + \Delta\dot{\theta}_f\,\frac{1}{6\Omega[1+\frac{1}{2}\mu^2]}\right]$$

II (2) (b) $$\left.\frac{\partial a'}{\partial b_1}\right|_{a_1 a_0} = -\frac{a\sigma a_0}{12 C_T}$$

(c) $$\left.\frac{\partial a'}{\partial a_1}\right|_{b_1 a_0} = 1 + \frac{a\sigma}{4C_T}\left[\frac{\lambda}{2} + \mu a_1 - \theta_0\mu^2\right]$$

Furthermore, from the expressions derived in Part I for the moment balance come the following:

$$\left.\frac{\partial M}{\partial a'}\right|_{a_1} = hT \qquad\qquad \left.\frac{\partial M}{\partial a_1}\right|_{a'} = \frac{ebM_s\Omega^2}{2}$$

Substitution of all of the expressions derived above into equation II (1) yields

$$\frac{\partial M}{\partial ()} = \left.\frac{\partial M}{\partial ()}\right|_{a_1 b_1 a_0} + \frac{\partial a_1}{\partial ()}\left\{ hT\left[1 + \frac{a\sigma}{4c_T}\left(\frac{\lambda}{2} + \mu a_1 - \theta_0 \mu^2\right)\right]\right.$$

II (3)

$$\left. + \frac{ebM_s\Omega^2}{2}\right\}$$

$$+ hT\left\{\frac{\partial a'}{\partial a_0}\frac{\partial a_0}{\partial ()} + \frac{\partial a'}{\partial b_1}\frac{\partial b_1}{\partial ()}\right\}$$

The last term in equation II (3) gives the net effect of coning and lateral flapping, and can be expanded as

$$\frac{hTa\sigma}{12\Omega C_T}\left[a_o \frac{\partial \dot{\Theta}_f}{\partial (\;)} + \frac{\Delta \dot{\Theta}_f}{\left[1+\frac{1}{2}\mu^2\right]} \frac{\partial a_o}{\partial (\;)} + \right.$$

$$\left. + 3\mu a_o \Omega \left(1 - \frac{9}{8} \frac{1}{\left[1-\frac{1}{2}\mu^2\right]}\right) \frac{\partial a_o}{\partial (\;)} \right]$$

When the variable to be inserted in the ( ) is $\dot{\Theta}$, the expression is approximately

$$\frac{hT}{12\Omega C_T} a_o$$

but for any other variable inserted in the ( ), the expression reduces to

$$\frac{hT}{4C_T} a_o \left[1 - \frac{9}{8} \frac{1}{\left[1-\frac{1}{2}\mu^2\right]}\right] \frac{\partial a_o}{\partial (\;)}$$

which is quite small in any condition. . The effect of lateral and collective flapping on the moment derivatives is thus seen to be small, especially when it is remembered from Part I that, for the high speed helicopter with a relatively large tail, most of the pitch damping comes from the tail. The last term in equation II (3) is considered zero and means that coning and lateral flapping cancel each other's influence on the longitudinal dynamics of the helicopter.

The result is that the moment derivatives can be written as the sum of two terms, those depending on longitudinal flapping and those not depending on any flapping. The flapping dependent contribution can further be classified as that part due to flapping hinge offset and that part due to the coincidence of R vector tilt with TPP tilt. The latter is the term whose magnitude is often overestimated by the R⊥TPP assumption, and whose importance will be investigated in the coming discussion.

It has been said that the coincidence of the R vector and the TPP motions is measured by

$$\frac{\partial a'}{\partial a_1} = 1 + \frac{a\sigma}{4C_T}\left[\frac{\lambda}{2} + \mu a_1 - \theta_0 \mu^2\right] \qquad \text{II (2c)}$$

At very low inflows and forward velocities it is easy to see that the R⊥TPP approximation is fairly good. As speed, power, and inflow increase, however, as they do with the newer high speed helicopters, the sign and magnitude of the sum within the bracket is not obvious. It can be said, however, that in a high power climb condition λ and θ_0 will both be large enough to assure that $\partial a'/\partial a_1 < 1$

whereas in a low power descent, autorotation for instance, λ and θ_0 will be small and

$$\partial a'/\partial a_1 > 1$$

In level flight it is necessary to expand the terms and make approximations to the expression in order to gain insight as to what are the important trends and contributions in terms of more fundamental parameters. The obvious sensitivity of the expression to trim conditions should instill caution when considering approximations. Usually good approximations are found to alter the magnitude and even the sign of the whole expression because the terms are all about the same size. Substituting into the bracketed part of equation II(2c) for a a_1 gives

$$\left[\frac{\lambda}{2} + \mu a_1 - \theta_0 \mu^2\right] = \frac{\lambda}{2} - \theta_0 \mu^2 + \frac{\mu^2}{\left[1 - \frac{1}{2}\mu^2\right]}\left[\frac{8}{3}\theta_0 + 2\lambda\right]$$

in which the approximation

$$\left[1 - \frac{1}{2}\mu^2\right] \doteq 1$$

is made (normally < 5%, error), and into which is substituted

$$\frac{\theta_0}{3} = \frac{\left[\frac{2C_T}{a\sigma} - \frac{\lambda}{2}\right]}{\left[1 + \frac{3}{2}\mu^2\right]}$$

to give

$$\left[\frac{\lambda}{2} + \mu a_1 - \theta_0 \mu^2\right] = \frac{10\mu^2 C_T}{a\sigma\left[1 + \frac{3}{2}\mu^2\right]} + \frac{\lambda}{2}\left[1 + \mu^2\left(4 - \frac{5}{\left[1 + \frac{3}{2}\mu^2\right]}\right)\right]$$

Finally, the approximation

$$\left[4 - \frac{5}{\left[1 + \frac{3}{2}\mu^2\right]}\right] << 1$$

is made (normally < 5% error) and the resulting expression substituted back into equation II (2c) to give

II (4) $$\frac{\partial a'}{\partial a_1} \doteq 1 + \left[\left(\frac{5}{2}\right)\frac{\mu^2}{\left[1 + \frac{3}{2}\mu^2\right]} + \frac{a}{8}\,\frac{\lambda}{(C_T/\sigma)}\right]$$

The inflow ratio, λ , is

$$\lambda = \frac{U_0 \sin \alpha_{ANF} - v_i}{\Omega R}$$

and is obviously negative in level flight and grows with forward speed, but the actual magnitude is sensitive to angle of attack and induced effects. Because of this the trend for a decrease in $\partial a'/\partial a_1$ with inflow is seen, but magnitudes in level flight are still not obvious. The values exhibited by the data helicopters are presented in figure (8) and will serve to characterize the "typical" trends for heavy, high speed helicopters. In table II (1) are the values found using the exact expression given in equation II , at 150 kts. The sensitivity is seen to be great, for the approximations are modest. The approximation does, however, exhibit the right sign and trends.

TABLE II (1)

HELICOPTER	$\frac{\lambda}{(C_T/\sigma)}$	$\frac{\partial a'}{\partial a_1}$	$\frac{\partial a'}{\partial a_1}$ (approx.)
42000 FWD	-1.5	.552	.195
24000 FWD	-1.16	.35	.441
35000 AFT	-1.26	.58	.37
18000 AFT	-2.24	-.04	-.31

The object of this analysis is to find what direct effects rotor flapping has on the stability derivatives of the helicopter, not to prove or disprove the validity of the R⊥TPP approximation. Because this is often a point of misunderstanding, however, it is considered worth while to dwell on the errors that would be incurred by the use of this approximation. These errors are of special concern to those who would devise some kind of helicopter stabilization system based on a reduction of rotor flapping. It will be shown that serious over-estimates of "improvement" are the result of this thinking.

When the R⊥TPP approximation is made, equation II (1) reduces to

$$\frac{\partial M}{\partial(\,)} = \left\{ hT + \frac{ebM_s\Omega^2}{2} \right\} \frac{\partial a_1}{\partial(\,)} + h\epsilon' \frac{\partial T}{\partial(\,)} + \left.\frac{\partial M}{\partial(\,)}\right|_{(\text{fuselage+tail})}$$

when it really should read

$$\frac{\partial M}{\partial(\,)} = hT \frac{\partial a'}{\partial(\,)} + \frac{ebM_s\Omega^2}{2} \frac{\partial a_1}{\partial(\,)} + h\epsilon \frac{\partial T}{\partial(\,)} + \left.\frac{\partial M}{\partial(\,)}\right|_{(\text{fuselage+tail})}$$

where ϵ' is computed with a_1 instead of a'. (The error incurred here is indicated by the plots of a' and a_1 in figure 9).

When the term $hT \dfrac{\partial a'}{\partial(\,)}$ is of significance to the whole expression,

then the approximation can cause errors. It was seen in Part I that the influence of the R vector tilt in $M_{\dot{\theta}}$ is small with high inflows, or power settings. The approximation thus causes little change in the value of $M_{\dot{\theta}}$ in these conditions. As the power settings are reduced, of course, the R vector tilt is more important, but the approximation becomes better for the same reasons. The derivatives Mw and Mu are the ones most affected. In these derivatives the "hT" term is of the same magnitude as the other terms, and the differences in $\dfrac{\partial a'}{\partial(\,)}$ and $\dfrac{\partial a_1}{\partial(\,)}$, become

large. Figures (3, 4 and 10) show the divergence between the two parameters, and indicate the magnitude of the errors. The static stability is over-estimated in both cases, the error increasing with speed.

The approximation does not appreciably effect any of the important force stability derivatives. Flapping has an influence in the X force derivatives, Xw and Xu, because of the influence of flapping on the inplane H force. These derivatives are of relative unimportance to this work and so the effects of flapping are considered to be only on the moment derivatives.

Having now seen how flapping influences the forces and moments, and thus the stability derivatives, it remains to be shown by how much the derivatives may be changed by controlling the amount of flapping - as with a flapping feedback system.

Longitudinal Flapping

Consider first an hypothetical perfect feedback system designed to keep longitudinal flapping zero in some unspecified manner. If this were an infinitely fast feedback loop such that $a_1 \equiv 0$, then in the expressions for the moment derivatives

$$\frac{\partial a_1}{\partial \alpha} = \frac{\partial a_1}{\partial u} = \frac{\partial a_1}{\partial \dot{\theta}} = \frac{\partial a'}{\partial a_1} = 0$$

The derivatives are changed by the now zero contributions from flapping hinge offset and the R vector tilt due to flapping. The moment derivatives are all reduced and the changes produce a different set of helicopter characteristics. It is to be noted that this is not a feedback to the cyclic control (which will be taken up later) but rather a computation of stability derivatives ignoring any contributions from longitudinal flapping. As such, it is now to be shown that the effective result, in the high inflow/speed condition, is helicopter stability and control characteristics similar to the same helicopter without flapping hinge offset. When equation II (3) is rewritten in a simple form,

II (5) $$M_u = \underbrace{\left\{ \underbrace{\frac{hT}{I_y}}_{\text{R vector tilt}} \frac{\partial a'}{\partial a_1} + \underbrace{\frac{ebM_s\Omega^2}{2I_y}}_{\text{hinge offset}} \right\} \frac{\partial a_1}{\partial ()}}_{\text{DUE TO FLAPPING}} + \underbrace{\frac{1}{I_y} \left.\frac{\partial M}{\partial ()}\right|_{a_1}}_{\text{NOT DUE TO FLAPPING}}$$

it can be seen that, if the first term in the bracket is small with respect to the second, the net effect of $\partial a_1 / \partial (\;) = 0$

is just the loss of the hinge offset term. Indeed, for the data helicopters in the high speed flight condition, where $\partial a' / \partial a_1$ is small, the following table shows the percentage of the first term to the total flapping contribution.

TABLE II (2)

	42000 FWD	35000 AFT	24000 FWD	18000 AFT
% (R tilt due to flapping) / (Total flap. cont.)	35%	28%	14%	0%

The hinge offset is included in the design of the helicopter primarily to increase the pilot's control power, or the amount of angular acceleration he can impart on the helicopter for a given stick deflection. If this perfect servo-system were installed, however, the pilot would lose the additional control power provided by the hinge offset, as well as that portion of the R vector tilt due to flapping. The pilot would feel almost as if he were flying the same helicopter but without hinge offset, especially for the last two entries in the table. This particular feedback thus seems to contradict the reasoning for designing the hinge offset in the first place.

This same type of contradiction will unfortunately arise in any feedback system design in which there is a simple mixing of the pilot inputs with feedback inputs. For example, consider an aircraft auto-pilot which can sense θ, $\dot{\theta}$, or $\ddot{\theta}$ and feed back the signal to sum with pilot inputs. If $\ddot{\theta}$ is fed back, the pilot is limited in how much torque he can apply about the aircraft center of gravity, but any gust induced torques are also limited. Similarly, for $\dot{\theta}$ and θ feedback, both the pilot and the gust disturbance are limited in how fast and how far, respectively, they can make the aircraft rotate.

It has been necessary to limit the scope of this work, and thus the reduced control problem associated with increased stability to gusts is not considered. Certainly, compromises will have to be made in any control system such that the pilot has the final authority in the system, if he wants it. In this work the pilot has pushed a "hold" button and desires to retain an equilibrium trimmed flight condition, and never enters a control input.

Along the same lines, it is very interesting to note that a negative $a_1 \rightarrow B_1$ feedback system (opposite sign of the one just considered), a so called "biased cyclic control system", has been investigated in reference (4) in connection with improving the control characteristics of a helicopter. With this negative (really positive in normal servo-convention) feedback, and with a suitable linkage, it was possible to reduce the control sensitivity (maximum pitching rate per unit stick deflection) by increasing the pitch damping at constant control power. It is mentioned in that reference that the feedback system seems to the pilot to be like adding hinge offset, confirming the assertion made in this work. Gust induced loads were not considered in that reference. It is felt that if they had been, the system would have been considered undesirable due to increases in static instability. There is little feasibility that that particular feedback system, in the high speed flight condition for helicopters with relatively large tails, will improve control characteristics because $M_{\dot{\theta}}$ is made up primarily by the tail contribution, as was seen in Part I.

At 150 Knots the following changes in the moment derivatives of the data helicopters are found due to the perfect, infinitely fast feedback loop.

TABLE II (3)

	42000 FWD		35000 AFT		24000 FWD		18000 AFT	
	$a_1 \neq 0$	$a_1 = 0$	$a_1 \neq 0$	$a_1 = 0$	$a_1 \neq 0$	$a_1 = 0$	$a_1 \neq 0$	$a_1 = 0$
M_u	.003	.0017	.0018	.0009	.002	.0009	.0009	.0003
$M_{\dot{\theta}}$	-.38	-.22	-.54	.386	-.767	-.583	-.53	-.41
M_{α}	.585	-.041	7.54	1.98	2.42	-.310	3.8	.31

The changes in the derivatives change the helicopter characteristic equation, and the resulting root locations are shown in figure (11) for all of the data helicopters at 150 kts. As can be seen from the changes in root locations, improvement in stability is not impressive, especially in the aft center of gravity cases.

The reason the improvements are not impressive is because the fuselage, tail, and non-flapping-dependent R vector tilt ($\partial a'/\partial a_1$ small) are large. This should be considered as typical of large, heavy, high speed helicopters, and will become more so as helicopter configuration approaches that of a compound helicopter. The truer this is, consequently, the less effective will be any type of helicopter stabilization system involving rotor flapping feedback control.

For the one case where this feedback has succeeded in changing the sign and magnitude of M_α enough to put the roots in an oscillatory region, it should be noted that the frequency of the mode is on the order of an airplane phugoid mode, and does not resemble the airplane classical "short period" mode in regard to frequency or damping ratio. The change in sign of the effective M_α for both forward C.G. cases does change the direction of the initial pitching, however, and would result in a decreased "local g" response that a pilot would feel.

The analysis presented above is for only one flight condition, at 150 kts, and shows that helicopter stability can be improved, however modestly, by reducing longitudinal flapping. It is clear that the improvement is a function of the relative changes in the moment derivatives, M_α and $M_{\dot\theta}$. Both are reduced by the reduced flapping, and the new root locations on the complex plane depend on which one is reduced "more". In other words, at each forward velocity the suppression of longitudinal flapping will change the stability characteristics by some amount, and that amount depends on the size of the flapping derivatives, $\partial a_1/\partial\dot\theta$ and $\partial a_1/\partial\alpha$, the relative amount of R tilt due to flapping, $\partial a_1'/\partial a_1$

and, finally, how much the rotor actually contributes to the stability derivative (e.g., the tail contributes the major portion of $M_{\dot\theta}$ at high speeds). These will all change with speed, and the plot of figure (12) shows that the relative changes in the derivatives for the "average" helicopter are such as to lend credence to the flapping feedback as speed is increased. Unfortunately, it has been shown that, even at the high speeds where, from figure (12), the feedback seems most desirable, improvement in stability is only modest.

Collective Flapping (Coning)

Near the beginning of Part II it was shown that Δa_1 and Δb_1 have little, indeed negligible, effect on the orientation of the rotor resultant force vector, R, except when the helicopter is pitching. In the absence of pitching, the Δa_o and Δb_1 contributions to the H force are almost equal and opposite in response to an angle of attack change. In a static sense it can then be said that coning angle has no direct influence on the stability derivatives. It is, however, acceptable to approximate a proportionality of thrust and collective pitch, which in turn, in the absence of induced effects, is nearly proportional to the coning angle. It is approximated as

$$\Delta T = \left.\frac{W}{a_o}\right|_{trim} \Delta a_o$$

and, if all of the lift comes from the thrust of the rotor,

$$Z_w = \frac{1}{m}\frac{\partial Z}{\partial w} \doteq -\left.\frac{W}{a_o}\right|_{trim}\frac{\partial a_o}{\partial w} \doteq \frac{g}{a_o}\frac{\partial a_o}{\partial w}$$

It is now possible to write a feedback equation, using this simple approximation, that says the collective pitch of the blades is proportional to the coning angle

$$\Delta \theta_{o_{A/P}} = -J\Delta a_o$$

If the feedback loop is infinitely fast there is some value, $1 \Big/ \left(\frac{\partial a_0}{\partial \theta_0}\right)$ of gain such that $\Delta a_0 \equiv 0$, and then $\partial a_0 / \partial w = 0$ which causes $Z_w = 0$.

In addition, now that the collective pitch can vary, there are changes in TPP tilt and R vector tilt due to the derivatives

$$\frac{\partial a_1}{\partial \theta_0} \quad \text{and} \quad \frac{\partial a'}{\partial a_1}\frac{\partial a_1}{\partial \theta_0}$$

It will be shown later, in the analysis of the actual feedback systems on the analog computer, that these changes in a_1 and a' are of prime importance when considering any kind of thrust feedback. They result in cross control (M_{θ_0}) coupling and can significantly alter the angular accelerations on the helicopter.

For the present discussion, only the effects of reduced Z_w will be considered. When $Z_w = 0$ the characteristic equation of the constant speed approximation becomes

$$s(s - M_{\dot{\theta}}) - M_\alpha = 0$$

where the vertical force equation now reduces to the identity, $\Delta\alpha = \Delta\theta$. There is one unstable root for any positive M_α. Physically, this is the equation of motion of a weather vane. The sign and magnitude of M_α depends on the position of the pivot point. If the pivot is far enough back M_α is positive, and the weather vane diverges in rotation when disturbed.

A reduction in Zw causes a decrease in damping and frequency of any aircraft. It also causes a reduction of initial normal acceleration. To the airplane, with its large negative M_{α}, a reduced damping and frequency of the short period oscillation is usually a small "cost" for an auto pilot designed to reduce Zw, and thus the initial load factor. For a helicopter with a positive M_{α}, however, a decreased Zw is had only at the expense of an increased rate of pitching divergence. With a positive M_{α} it is mandatory to reduce the angle of attack quickly in order to achieve pitching stability, but this means increased load factors. This is clearly one of those cases in which other restrictions must be considered when deciding which is best.

Two Degrees Vs. Three Degrees of Freedom

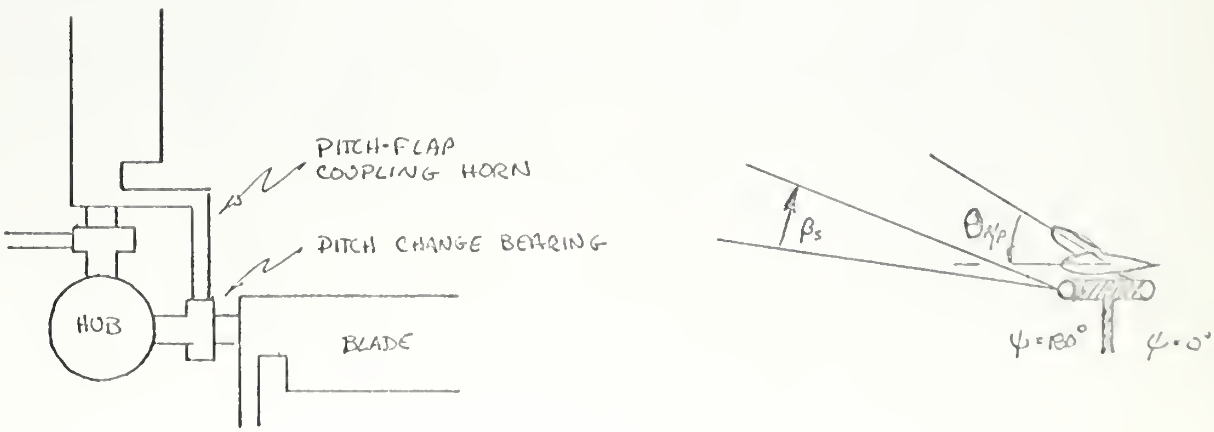
Before going on with the analysis of any particular feedback system, it is of interest to see how the perfect flapping feedback systems considered above affect the long period oscillation, or the mode of motion arising when forward speed is allowed to vary. Using the approximation derived in Part I, the roots for the 42000 lb helicopter have been computed for both the unmodified and the modified cases and are shown in figure (13). The root locations show a modest improvement in stability. The figure is not a conventional root loci, but rather the constant speed roots were computed for the two cases individually, and a root loci plotted for increasing Mu.

It is seen from the small change in long period roots that there is very little effect of flapping in the long period oscillation. From this observation it seems reasonable to consider only the constant speed approximate equations of motion for a simplified analysis. It is probable that no improvement in helicopter stability characteristics can be made without improvement in the "$\Delta u \equiv 0$ mode", and so this approximation will be used in the analytical portion of the remainder of this work. The analog computer results will show the complete degrees of freedom, however.

Oehmichen and Delta Three Pitch Flap Coupling

Flapping feedback control can be implemented on a helicopter mechanically or electronically. Linkages or sensors are used in some way to sense blade flapping and alter the pitch setting of the blades cyclically or collectively. There are two mechanical schemes which deserve special interest. The first is Oehmichen pitch flap coupling because, from an analysis of this mechanical device, insight into other more complex systems currently being proposed for helicopter stabilization can be found. The second is Delta Three pitch flap coupling because it has been considered as a possible means of achieving blade and helicopter stability for many years, but has not, to the author's knowledge, been investigated in connection with the dynamics of the helicopter as a whole. The two mechanical schemes are described below.

With Oehmichen pitch flap coupling the cyclic variation of pitch that a blade "sees" (excluding any pilot inputs) is proportional to the flapping angle of a blade (in the case of a four bladed rotor, but otherwise this should be the angle of the TPP) 90° in front of it (in the direction of rotation). This is seen in the figure below for a four bladed rotor.



POSITIVE OEHMICHEN

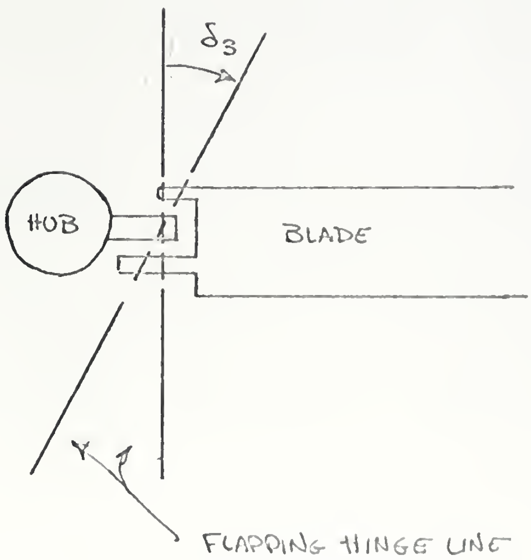
Positive Oehmichen coupling is defined here as:

$$\theta_0 + B_{1s}\cos\psi + A_{1s}\sin\psi = -H\left\{a_0 - a_1\cos\psi - b_1\sin\psi\right\}$$

In words, positive blade flapping resulting in a decrease in pitch of the blade 90° behind it (counter to the direction of rotation) is defined as positive. The mechanical gearing ratio is the gain H.

The Delta Three hinge is the blade flapping hinge and when the hinge is cocked opposite to the direction of rotation it is said that $\delta_3 = (\measuredangle$ between the hinge line and the blade transverse axis). It is shown positive in the figure.

POSITIVE DELTA THREE



With positive Delta Three, when a blade flaps up it reduces its own pitch angle by tan δ_3.

Thus $$\Delta\theta = -\tan\delta_3 \, \Delta\beta$$

or

$$\theta_0 - A_{1s}\cos\psi - B_{1s}\sin\psi = -\tan\delta_3 \left\{ a_0 - a_1\cos\psi - b_1\sin\psi \right\}$$

It is seen that, with both schemes the collective pitch is decreased proportional to the coning angle, but that the two devices differ in the cyclic component of feedback. The cyclic pitch due to the feedback, B_{1s} A/P is proportional to longitudinal flapping in the Oehmichen system, but proportional to lateral flapping in the Delta Three system.

In order to effectively compare the two systems it is convenient to separate the coning and cyclic components. This results in the feedback equations:

OEHMICHEN	DELTA THREE
$\Delta\theta_0 = -J\Delta a_0$	$\Delta\theta_0 = -J\Delta a_0$
$\Delta B_{1s} = K\Delta a_1$	$\Delta B_{1s} = -K_\delta \Delta b_1$
$\Delta A_{1s} = K\Delta b_1$	$\Delta A_{1s} = -K_\delta \Delta a_1$

where J = K = H, and J = K_δ = tan δ_3 are the conventional, mechanical arrangements for the two systems.

This separation in the feedback signals can be accomplished by suitable linkages or even by separate mechanical devices (reference 4), but, without regard for practicality, it is easier to think of the feedback as electrical signals originating from potentiometers mounted at the flapping hinges measuring blade flapping angles. In this way the signals may be used to drive servo-motors or hydraulic servos to vary the pitch of the blades at any azimuth position. The signals from different blades are easily combined and thus separation of the cyclic and collective components is possible. It is also possible with this kind of arrangement to integrate the feedback signals and, with suitable sensors, to create feedback signals proportional to the derivatives of blade flapping.

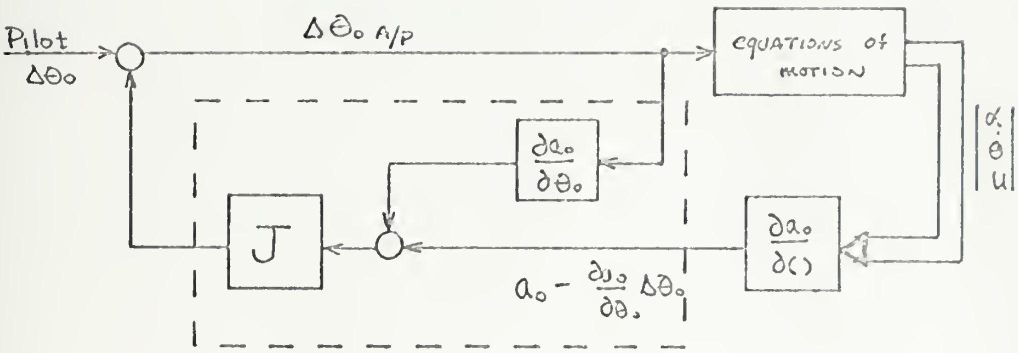
Flapping feedback can be introduced into the equations of motion on the analog computer very easily when the blade flapping variables are treated as they are in this work. On the analog computer diagram, figure (1), it is seen that the flapping variables are just different combinations of the helicopter variables, each multiplied by a different constant. In actuality the different feedbacks are just unique combinations of angle of attack, airspeed, and pitching rate. As such, the feedback equations can be written as:

$$\Delta\theta_0 = -J\Delta a_0 = -J\left\{ \frac{\partial a_0}{\partial w}\Delta w + \frac{\partial a_0}{\partial u}\Delta u + \frac{\partial a_0}{\partial \dot\theta}\Delta\dot\theta + \frac{\partial a_0}{\partial B_1}\Delta B_1 + \frac{\partial a_0}{\partial \theta_0}\Delta\theta_0 \right\}$$

II (6) $$\Delta B_{1s} = K\Delta a_1 = K\left\{ \frac{\partial a_1}{\partial w}\Delta w + \frac{\partial a_1}{\partial \dot\theta}\Delta\dot\theta + \frac{\partial a_1}{\partial u}\Delta u + \frac{\partial a_1}{\partial B_1}\Delta B_1 + \frac{\partial a_1}{\partial \theta_0}\Delta\theta_0 \right\}$$

$$\Delta B_{1s} = -K\Delta b_1 = -K\left\{ \frac{\partial b_1}{\partial w}\Delta w + \frac{\partial b_1}{\partial u}\Delta u + \frac{\partial b_1}{\partial \dot\theta}\Delta\dot\theta + \frac{\partial b_1}{\partial B_1}\Delta B_1 + \frac{\partial b_1}{\partial \theta_0}\Delta\theta_0 \right\}$$

It should be noticed that the control variables appear on both sides of the equations, and there is therefore a closed loop effect which determines the actual "gain" of the system. This is illustrated in a simplified block diagram for the collective feedback.



The portion outlined by the dotted line can be replaced by a single "block" which is the effective closed loop gain. One other observation that should be made here is that the Delta Three and Oehmichen cyclic components are of opposite sign. $B_{1S\,A/P}$ is increased when the rotor flaps back (positive flapping) with positive Oehmichen, and $B_{1S\,A/P}$ is decreased when the rotor tilts to the advancing side (positive flapping) with positive Delta Three.

The feedback equations can be rewritten in terms of effective gains which take into account the closed loop effects of the dependence of flapping on the control inputs. Derivation of these effective gains is shown for the Delta Three system. Equations II (6) can be rewritten as

II (7) $$\Delta \Theta_0 = \frac{-J}{\left[1 + \frac{\partial a_0}{\partial \theta_0} J\right]} \left\{ \frac{\partial a_0}{\partial w} \Delta w + \cdots \frac{\partial a_0}{\partial B_{1s}} \Delta B_{1s} \right\}$$

II (8) $$\Delta B_{1s} = \frac{-K_\delta}{\left[1 + \frac{\partial b_1}{\partial B_{1s}} K_\delta\right]} \left\{ \frac{\partial b_1}{\partial w} \Delta w + \cdots \frac{\partial b_1}{\partial \theta_0} \Delta \Theta_0 \right\}$$

Defining

$$K_\delta' \equiv \frac{K_\delta}{\left[1 + \frac{\partial b_1}{\partial B_{1s}} K_\delta\right]} \qquad J' \equiv \frac{J}{\left[1 + \frac{\partial a_0}{\partial \Theta_0} J\right]}$$

and substituting equations II (7) and II (8) into each other results in:

II (9)
$$\Delta\Theta_0 = -J''\left\{\left[\frac{\partial a_0}{\partial w} - K_\delta'\frac{\partial a_0}{\partial B_{1s}}\frac{\partial b_1}{\partial w}\right]\Delta w + \left[\frac{\partial a_0}{\partial \dot\theta} - K_\delta'\frac{\partial a_0}{\partial B_{1s}}\frac{\partial b_1}{\partial \dot\theta}\right]\Delta\dot\theta + \left[\frac{\partial a_0}{\partial u} - K_\delta'\frac{\partial a_0}{\partial B_{1s}}\frac{\partial b_1}{\partial u}\right]\Delta u\right\}$$

II (10)
$$\Delta B_{1s} = -K_\delta''\left\{\left[\frac{\partial b_1}{\partial w} - J'\frac{\partial b_1}{\partial \theta_0}\frac{\partial a_0}{\partial w}\right]\Delta w + \left[\frac{\partial b_1}{\partial \dot\theta} - J'\frac{\partial b_1}{\partial \theta_0}\frac{\partial a_0}{\partial \dot\theta}\right]\Delta\dot\theta + \left[\frac{\partial b_1}{\partial u} - J'\frac{\partial b_1}{\partial \theta_0}\frac{\partial a_0}{\partial u}\right]\Delta u\right\}$$

where

$$J_\delta'' = \frac{J'}{\left[1 - K_\delta' J'\frac{\partial b_1}{\partial \theta_0}\frac{\partial a_0}{\partial B_{1s}}\right]} \qquad K_\delta'' = \frac{K_\delta'}{\left[1 - K_\delta' J'\frac{\partial b_1}{\partial \theta_0}\frac{\partial a_0}{\partial B_{1s}}\right]}$$

The Oehmichen feedback equations are analogous. They are:

$$\text{II (11)} \quad \Delta\Theta_o = -J''\left\{\left[\frac{\partial a_o}{\partial w} + K'\frac{\partial a_o}{\partial B_{1s}}\frac{\partial a_1}{\partial w}\right]\Delta w + \cdots\right\}$$

$$\text{II (12)} \quad \Delta B_{1s} = K''\left\{\left[\frac{\partial a_1}{\partial w} - J'\frac{\partial a_1}{\partial \theta_o}\frac{\partial a_o}{\partial w}\right]\Delta w + \cdots\right\}$$

where

$$J' \equiv \frac{J}{\left[1 + \frac{\partial a_o}{\partial \theta_o} J\right]} \qquad K' \equiv \frac{K}{\left[1 - \frac{\partial a_1}{\partial B_{1s}} K\right]}$$

$$J'' \equiv \frac{J'}{\left[1 + K'J'\frac{\partial a_o}{\partial B_{1s}}\frac{\partial a_1}{\partial \theta_o}\right]} \qquad K'' \equiv \frac{K'}{\left[1 + K'J'\frac{\partial a_o}{\partial B_{1s}}\frac{\partial a_1}{\partial \theta_o}\right]}$$

Equations II (9) - II (12) are the complete feedback equations written in such a form that they can be introduced into the equations of motion, equation (1), as control inputs. They are multiplied by the respective control derivatives (e.g. M_{Θ_o}, M_{B_1}, etc.) and transferred to the left hand side of equation (1). This procedure results in what may be thought of as modified stability derivatives. It is shown for the constant speed moment equation modified by Oehmichen feedback. From equation (1),

$$-M_w \Delta w + (s - M_{\dot\theta})\Delta\dot\theta = M_w \Delta w_g + M_{\Theta_o}\Delta\Theta_o + M_{B_1}\Delta B_{1s}$$

which can be modified by replacing $\Delta\Theta_o$ and ΔB_{1s} by equations II (11) and II (12), resulting in:

$$-M_w \Delta w_a + (s - M_{\dot{\theta}})\Delta\dot{\theta} = M_w \Delta w_g + M_{\theta_o}\left[\text{eqn II}(11)\right] + M_{B_1}\left[\text{eqn II}(12)\right]$$

Rearranging gives:

$$M_w' \Delta w_a + (s - M_{\dot{\theta}}')\Delta\dot{\theta} = M_w' \Delta w_g$$

where

$$M_w' \equiv M_w - M_{\theta_o} J''\left[\frac{\partial a_o}{\partial w} + K'\frac{\partial a_o}{\partial B_1}\frac{\partial a_1}{\partial w}\right] + M_{B_1} K''\left[\frac{\partial a_1}{\partial w} - J'\frac{\partial a_1}{\partial \theta_o}\frac{\partial a_o}{\partial w}\right]$$

$$M_{\dot{\theta}}' \equiv M_{\dot{\theta}} - M_{\theta_o} J''\left[\frac{\partial a_o}{\partial \dot{\theta}} + K'\frac{\partial a_o}{\partial B_1}\frac{\partial a_1}{\partial \dot{\theta}}\right] + M_{B_1} K''\left[\frac{\partial a_1}{\partial \dot{\theta}} - J'\frac{\partial a_1}{\partial \theta_o}\frac{\partial a_o}{\partial \dot{\theta}}\right]$$

The procedure is identical for all of the derivatives and for each proportional feedback system. It results in equations of motion of exactly the same form as without feedback, but with modified derivatives denoted by the (') prime. The writing of all of the modified derivatives is omitted here because little insight can be gained by looking at such complicated expressions whose terms may all be of the same size.

It is appropriate to analize each component of the two feedback systems separately before considering the complete feedback equations. There are three variations; lateral, longitudinal, or collective flapping may be used as the feedback "signal", independently of each other.

Longitudinal (Oehmichen) Cyclic Feedback

The feedback equation for this system is written as

$$\Delta B_{1s} = K \Delta a_1 \qquad \Delta\theta_o \equiv 0$$

$$\Delta B_{1s} = K'\left[\frac{\partial a_1}{\partial w}\Delta w + \frac{\partial a_1}{\partial \dot\theta}\Delta\dot\theta + \frac{\partial a_1}{\partial u}\Delta u\right]$$

which can be substituted into the equations of motion and then, regrouping, results in modified stability derivatives as follows.

$$M_w' = M_w + M_{B_1}K'\frac{\partial a_1}{\partial w} \qquad Z_w' = Z_w + Z_{B_1}K'\frac{\partial a_1}{\partial w}$$

$$M_{\dot\theta}' = M_{\dot\theta} + M_{B_1}K'\frac{\partial a_1}{\partial \dot\theta} \qquad U_o' = U_o + Z_{B_1}K'\frac{\partial a_1}{\partial \dot\theta}$$

M_{B_1}, Zw, and $\partial a_1/\partial\dot\theta$ are always negative, Z_{B_1} and $\partial a_1/\partial w$ are always positive. If Mw is positive, as for the unstabilized helicopter, then all of the primed derivatives are smaller than the unmodified ones. The resulting characteristic equation of motion is similar to the $a_1 \equiv 0$ case previously considered, but with two important differences.

First, the "autopilot" control is now being accomplished by tilting the control axis in response to a change in longitudinal flapping. In the hypothetical $a_1 \equiv 0$, it should be remembered, no provision is made for the tilt of the control axis - the flapping is "suppressed" in some unspecified manner, and any forces or moments dependent on flapping never even appear (see equation II (5)). In this actual $a_1 \rightarrow B_{1S}$ feedback case, however, the control axis, ANF, is tilted as the "autopilot" response to flapping caused by a gust disturbance. The change of ANF angle of attack, $\Delta\alpha_{ANF}$, changes the H force, thrust, and induces flapping. All of these

then change the moments on the helicopter. The moment derivatives are changed by

II (13)
$$M'_{(\,)} = M_{(\,)} + M_{B_1} K' \frac{\partial a_1}{\partial (\,)}$$

where M_{B_1} can be expressed as:

II (14)
$$M_{B_1} = -\frac{h}{I_y}\left\{ T\left(1 + \frac{\partial a'_1}{\partial \alpha}\right) + \epsilon \frac{\partial T}{\partial \alpha} \right\} - \frac{c b M_s \Omega^2}{2 I_y}\left(1 + \frac{\partial a_1}{\partial \alpha}\right)$$

By regrouping in equation II (14) to expose the implicit dependence on flapping, and substituting back into equation II (13), an expanded expression is found for a moment stability derivative.

II (15) $$M'_{(\,)} = M_{(\,)} - K\frac{\partial a_1}{\partial(\,)}\left\{\underbrace{\frac{hT}{I_y\left[1+K\frac{\partial a_1}{\partial\alpha}\right]}}_{\text{THRUST TILT}}\right.$$

$$+\underbrace{\frac{hT}{I_y}\left.\frac{\partial a'}{\partial\alpha}\right|_{a_1}\frac{1}{\left[1+K\frac{\partial a_1}{\partial\alpha}\right]}}_{\text{R tilt INDEPENDENT of flapping}} \quad + \underbrace{\frac{h\epsilon}{I_y}\frac{\partial T}{\partial\alpha}\frac{1}{\left[1+K\frac{\partial a_1}{\partial\alpha}\right]}}_{\text{R tilt IN TRIM}}$$

$$\left.+\underbrace{\frac{hT}{I_y}\frac{\partial a'}{\partial a_1}\frac{\partial a_1}{\partial\alpha}\frac{1}{\left[1+K\frac{\partial a_1}{\partial\alpha}\right]}}_{\text{flapping INDUCED R tilt}} \quad + \underbrace{\frac{ebM_s\Omega^2}{2I_y}\frac{\left[1+\frac{\partial a_1}{\partial\alpha}\right]}{\left[1+K\frac{\partial a_1}{\partial\alpha}\right]}}_{\text{flapping dependent hub MOMENT}}\right\}$$

It is seen that longitudinal (Oehmichen) cyclic feedback does not simply relieve hub moments, but influences the moments on the helicopter due to each term in the above expression. It does not necessarily completely relieve hub moments, whereas the hypothetical $a_1 \equiv 0$ case, earlier considered, does.

The second point of importance is the occurrence of the cross control derivative, Z_{B_1}, in the expressions for the modified derivatives. This causes further differences between this actual feedback and the hypothetical $a_1 = 0$ case. The cyclic control, B_{1S}, is primarily a moment producing control, but vertical forces are also produced by this control input, and can change the characteristics of the control response if the cross control derivative, Z_{B_1}, is large. An expression for Z_{B_1} in initially level flight is

$$\frac{\partial Z}{\partial B_1} = \frac{\partial T}{\partial \alpha}\left[1 - a'\alpha_0\right] - T\left[\alpha_0 \frac{\partial a'}{\partial \alpha} + a'\right]$$

but can be considerably simplified by neglecting all but the $\partial T/\partial \alpha$ term, which is a good approximation in level flight. The derivative then becomes

$$Z_{B_1} = \frac{1}{m}\frac{\partial T}{\partial \alpha} = -Z_\alpha = -U_0 Z_w$$

Z_{B_1} is hardly negligible in high speed flight where Z_w is on the order of unity. When the approximation is substituted into the expression for Z_w',

$$Z_w' = Z_w\left(1 - K'\frac{\partial a_1}{\partial \alpha}\right)$$

For all positive K the derivative is reduced, and thus the static normal loads on the helicopter will be reduced, due to cyclic feedback. The cross control causes little change in the "effective speed", U_0', in high speed flight.

In figures (14) are shown the root locations for each data helicopter with $a_1 \rightarrow B_{1S}$ feedback gain of unity (top part of each figure), including the effects of Z_{B_1}. (The lines between roots are not conventional root loci.) It is seen that little difference between this and the $\Delta a_1 \equiv 0$ case shows up on the complex plane. Figures (15) and (16) show analog computer responses for the 42000 lb helicopter with increasing gain, K, @ $Z_{B_1} = 0$; and for varying Z_{B_1} @ constant K. Figure (16) shows responses for K = .5, and K = 1.0, with Z_{B_1} at the value for the 42000 lb helicopter.

The most important observation to be made concerning the analog computer responses is the change in sign and magnitude of the initial $\ddot{\theta}$ response - signifying a changed M'_α. The helicopter pitches down like an airplane with high cyclic feedback gain, and is thus statically stable with angle of attack. It is also possible to see that the static normal load factor is reduced slightly when Z_{B_1} is increased from zero. Finally, it is apparent that cyclic feedback tends to stabilize the helicopter in the longitudinal plane, but, for all of the same reasons above, the same feedback causes a destabilizing lateral-longitudinal coupling term. This is because the derivative, $\partial b_1 / \partial \alpha$ is positive, causing positive lateral flapping in response to a vertical gust. When positive lateral cyclic feedback is employed, $\Delta A_{1S\,A/P} = K \Delta b_1$, the sign of the feedback is such as to increase the longitudinal flapping instead of reducing it, in response to a vertical gust.

Lateral (Delta Three) Cyclic Feedback

The cyclic component of Delta Three pitch flap coupling works the same way as the longitudinal feedback just considered, but the sign and magnitude of the feedback "signal" are different. The sign is different because a rotor tilts with a positive b_1 due to a vertical gust, and positive Delta Three then causes a decreased longitudinal cyclic, ΔB_{1S} A/D. This is clearly the wrong sign for longitudinal static stability. It is the right sign, however, for blade static stability. Right from the outset it is apparent that the positive Delta Three cyclic component will increase helicopter longitudinal static instability, and should be avoided from the point of view of this work. An investigation of negative Delta Three is warranted from this observation, and positive Delta Three will be disregarded until the collective component is included in the analysis.

The procedure for analyzing this feedback is identical to the longitudinal one. The modified stability derivatives that result can be written as (for the constant speed approximation):

$$M_w' = M_w - K_\delta' M_{B_1} \frac{\partial b_1}{\partial w} \qquad Z_w' = Z_w - K_\delta' Z_{B_1} \frac{\partial b_1}{\partial w}$$

$$M_{\dot\theta}' = M_{\dot\theta} - K_\delta' M_{B_1} \frac{\partial b_1}{\partial \dot\theta} \qquad U_0' = U_0 - K_\delta' Z_{B_1} \frac{\partial b_1}{\partial \dot\theta}$$

When negative Delta Three is used, all of the derivatives are reduced. Their magnitudes differ from the Oehmichen feedback only by the magnitudes of $\partial b_1/\partial w$ and $\partial b_1/\partial \dot\theta$, for the same mechanical gains.

It is possible to compare the two types of feedback by comparing the flapping derivatives. It can be shown that

$$\frac{\partial a_1}{\partial w} = \frac{9}{\gamma} \frac{\left[1 + \frac{\mu^2}{2}\right]}{\left[1 - \frac{\mu^2}{2}\right]} \frac{\partial b_1}{\partial w}$$

and

$$\frac{\partial a_1}{\partial \dot{\theta}} = \frac{16}{\gamma} \frac{\partial b_1}{\partial \dot{\theta}}$$

For normal values of γ (11-13) it is seen that

$$\frac{\partial b_1}{\partial w} > \frac{\partial a_1}{\partial w} \qquad\qquad \frac{\partial a_1}{\partial \dot{\theta}} > \frac{\partial b_1}{\partial \dot{\theta}}$$

and, as forward speed increases, $\frac{\partial b_1}{\partial w}$ approaches $\frac{\partial a_1}{\partial w}$.

The similarity of the two types of feedback is evident, but the reduction in pitch damping is smaller, and the increase in static angle of attack stability is greater with the negative Delta Three feedback. This is seen in figure (17) where analog computer traces of helicopter motions with the different feedbacks is shown. The positive Delta Three arrangement is clearly destabilizing, and the negative Delta Three is clearly "better" than the Oehmichen feedback. In fact, the cyclic component of negative Delta Three feedback alone would seem to be quite a good stabilization device for a helicopter, on the basis of these results.

Collective (Coning) Feedback

The collective feedback equation can be written

$$\Delta\theta_0 = -J \ \Delta a_0 \qquad\qquad \Delta B_{1s} \equiv 0$$

and it should be noted that there is no distinction between positive Delta Three and Oehmichen because $\Delta B_{1s} \equiv 0$.

When the feedback is introduced into the equations of motion, as before, and the approximation that

$$\frac{\partial a_0}{\partial w} >> \frac{\partial a_0}{\partial \dot{\theta}}$$

is made ($\frac{\partial a_0}{\partial \dot{\theta}}$ arises only due to the change in local linear velocity at the hub when pitching), the modified stability derivatives become

$$M_w' = M_w - M_{\theta_0} J' \frac{\partial a_0}{\partial w} \qquad\qquad Z_w' = Z_w - Z_{\theta_0} J' \frac{\partial a_0}{\partial w}$$

$$M_{\dot{\theta}}' \doteq M_{\dot{\theta}} \qquad\qquad U_0' \doteq U_0$$

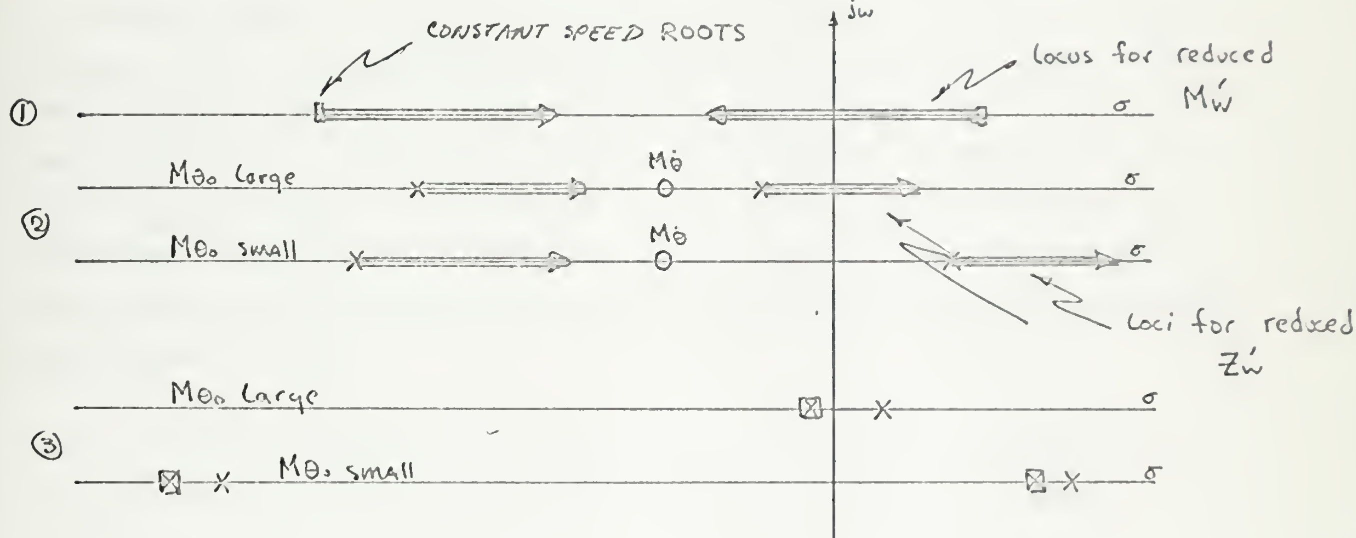
It is seen that, in addition to the anticipated result that Zw' is reduced, Mw' is also reduced (made more statically stable) if M_{θ_0} is positive. An expression for M_{θ_0} can be written as

$$\frac{\partial M}{\partial \theta_0} \doteq h\left\{ \frac{\partial T}{\partial \theta_0}\epsilon + T\frac{\partial a'}{\partial \theta_0} \right\} + \frac{ebM_s\Omega^2}{2}\frac{\partial a_1}{\partial \theta_0}$$

where $\partial a_1/\partial \theta_0$ and $\partial a'/\partial \theta_0$ are both always positive. The sign of the remaining term depends on the orientation of the R vector in trim. For the data helicopters the second two terms are always larger than the first (the first term is generally negative because a negative ϵ results in a stable contribution to M_α), resulting in positive M_{θ_0}. It can also be said that, unless the horizontal tail, fuselage, and hub contribute excessive nose up pitching moments in trim, M_{θ_0} must be positive and will become increasingly larger with forward speed. For the data helicopters M_{θ_0} increases by an order of magnitude from 60 to 150 knots.

Positive coning angle feedback thus reduces both Zw' and Mw' for the helicopters represented in this study, resulting in a reduced maximum normal acceleration due to a gust disturbance, and an increase in angle of attack static stability. These changes cause changes in the helicopter dynamics as represented by shifts of the characteristic roots on the complex plane. As has been seen before, the reduced M_α' will "improve", and the decreased Zw' will vitiate the stability of the helicopter. The net effect will, of course, depend on the relative sizes of M_α, Zw, Z_{θ_0}, and, especially, M_{θ_0}.

In the following figure this point is illustrated by showing the various positions of the roots as M_w' and Z_w' are reduced, and the resulting positions of the roots when M_{θ_0} is large and small.



① $\Delta u = 0$ ROOTS UNMODIFIED BY FEEDBACK
Roots of ① are poles of ② with new value of M_w'.

③ FINAL ROOT LOCATIONS for different combinations of M_{θ_0} and Z_{θ_0}.
X is for Z_{θ_0} large, $\boxed{X}$ is for Z_{θ_0} small.
Shows effect of Z_{θ_0} is much smaller than effect of M_{θ_0}.

The resulting root positions are thus very dependent on M_{θ_0}, the larger M_{θ_0} is, the greater the improvement in stability.

In figure (14) (bottom part of each entry) are shown the root locations for the data helicopters when J = 1. The very different ways in which the feedback has changed the root locations for the four helicopters attests to the sensitivity of the coning feedback to configuration. The 42000 lb helicopter exhibits the most pronounced change in that the helicopter is made stable with coning feedback. The analog computer responses of figure (18) show the dependence of this helicopter on M_{θ_0}. When M_{θ_0} is zero, the helicopter is divergent, but with M_{θ_0} set equal to its calculated value on the analog computer, the helicopter response is considerably improved with coning angle feedback. This is the only helicopter of the four, however, in which an appreciable improvement is shown.

In order to determine the magnitude of the modified derivatives, J' as a function of J, the mechanical gain must be known. According to the way this is defined,

$$J' = \frac{J}{\left[1 + \frac{\partial a_0}{\partial \theta_0} J\right]}$$

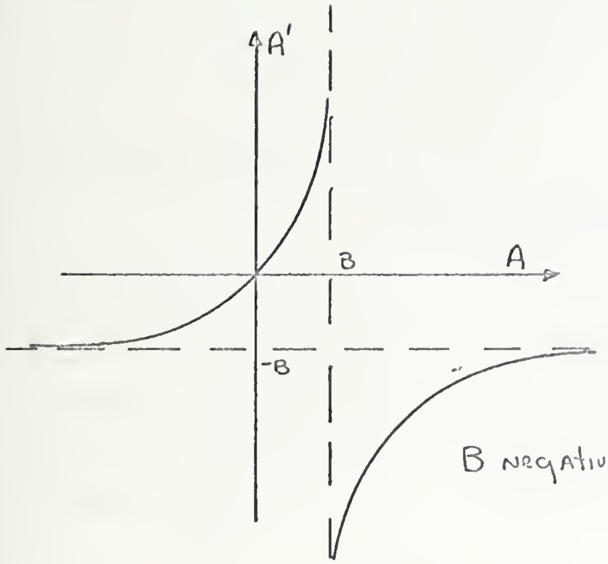
and thus at every different flight condition (different $\frac{\partial a_0}{\partial \theta_0}$)

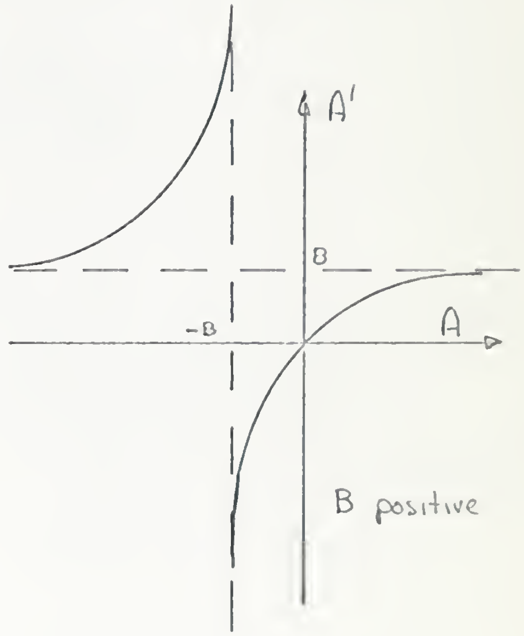
the actual "closed loop" gain is different.

The function has the form (as do all of the "closed loop" gains in these feedback systems)

$$A' = \frac{A}{\left[1 + A/B\right]}$$

and a sketch of this function is shown below





The function is linear thru zero, and if B is large, an approximately linear region may be assumed for small A, and then $A' \doteq A$.
When B is small, positive A' is "gain limited", but negative A' is amplified rapidly toward infinity.

In this case, $J = A$ and $B = 1 \Big/ \frac{\partial a_0}{\partial \theta_0}$, and it is thus necessary to investigate the nature of $\partial a_0 / \partial \theta_0$.

From the expression given for a_0,

$$\frac{\partial a_0}{\partial \theta_0} = \frac{\gamma}{8} \left[1 + \mu^2 + \frac{4}{3} \frac{\partial \lambda}{\partial C_T} \frac{\partial C_T}{\partial \theta_0} \right]$$

In high speed flight $\partial \lambda / \partial C_T$ approaches zero (negatively) due to the reduced importance of induced effects. In the flight condition of interest it is reasonable to assume

$$1 >> \left[\mu^2 + \frac{4}{3} \frac{\partial \lambda}{\partial C_T} \frac{\partial C_T}{\partial \theta_0} \right]$$

and a high speed estimate is

$$\partial a_0 / \partial \theta_0 \doteq \gamma / 8 \doteq 3/2 \quad \text{for } \gamma \doteq 12 .$$

From the sketch of the function for the "closed loop" gain it is seen that J' never gets bigger than about (+) 2/3, but approaches negative infinity even with very small negative gain. When J' is, in fact, approaching negative infinity, Z_w is approaching positive infinity and the roots of the characteristic equation are approaching the zeroes of the equation

$$(s + a)(s + b) - \Delta Z_w' (s - M_{\dot{\theta}}) = 0$$

when $M_{\theta_0} = 0$. This would seem to prove that negative coning feedback is good, but it is only because of an omission of the effects of the coning feedback on the effective angle of attack stability, M_w. When M_{θ_0} is not exactly zero, M_w is approaching positive infinity and thus negative coning feedback is impracticable.

In response to a vertical gust, the static response of the helicopter is

$$\Delta w_a = Z_w \Delta w_g + Z_{\theta_o} \Delta\theta_o + Z_{B_1} \Delta B_1$$

$$\Delta \dot{\dot{\theta}} = M_w \Delta w_g + M_{\theta_o} \Delta\theta_o + M_{B_1} \Delta B_1$$

or

$$\frac{\Delta \dot{w}_a}{\Delta w_g} = Z_w' = Z_w + Z_{\theta_o} \frac{\Delta\theta_o}{\Delta w_g} + Z_{B_1} \frac{\Delta B_1}{\Delta w_g}$$

II (16)

$$\frac{\Delta \dot{\dot{\theta}}}{\Delta w_g} = M_w' = M_w + M_{\theta_o} \frac{\Delta\theta_o}{\Delta w_g} + M_{B_1} \frac{\Delta B_1}{\Delta w_g}$$

where, from equations II (9) thru II (12) $\frac{\Delta\theta_o}{\Delta w_g}$ and $\frac{\Delta B_1}{\Delta w_g}$ can be expressed as:

OEHMICHEN — DELTA THREE

a) $$\frac{\Delta B_{1s}}{\Delta w_g} = K''\left[\frac{\partial a_1}{\partial w} - J'\frac{\partial a_1}{\partial \theta_o}\frac{\partial a_o}{\partial w}\right] \qquad \frac{\Delta B_{1s}}{\Delta w_g} = -K_\delta''\left[\frac{\partial b_1}{\partial w} - J'\frac{\partial b_1}{\partial \theta_o}\frac{\partial a_o}{\partial w}\right]$$

II (17)

b) $$\frac{\partial \theta_o}{\partial w_g} = -J''\left[\frac{\partial a_o}{\partial w} + K'\frac{\partial a_o}{\partial B_{1s}}\frac{\partial a_1}{\partial w}\right] \qquad \frac{\Delta\theta_o}{\Delta w_g} = -J_\delta''\left[\frac{\partial a_o}{\partial w} - K_\delta'\frac{\partial a_o}{\partial B_{1s}}\frac{\partial b_1}{\partial w}\right]$$

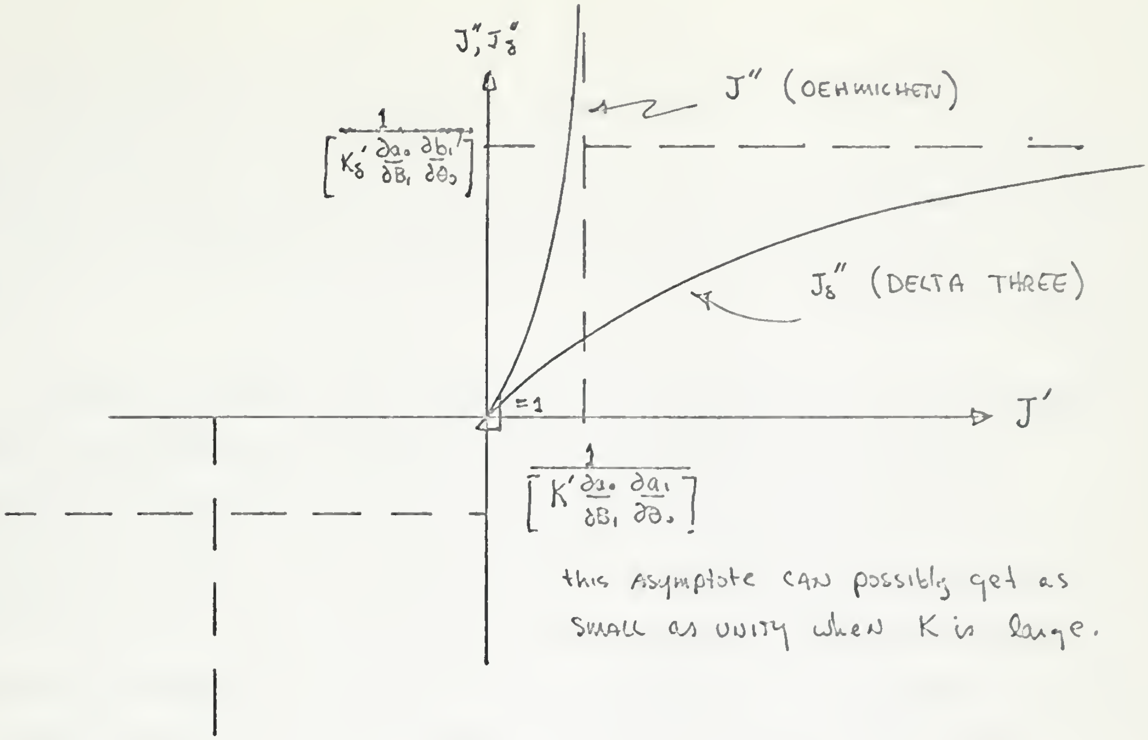
Figure (14) has already been referred to, but now will serve as a summary of the possibilities of longitudinal or collective flapping feedback used individually. It shows that, for gains of unity, improvements in stability are not impressive, and certainly not acceptable when the complexity of the mechanisms are considered. The analysis was not carried beyond gains of unity because it has been shown that there are blade motion stability limits at, or near, unity for both kinds of pitch flap coupling. References (3 and 6). These limits are the result of fixed shaft investigations of the dynamics "within" the rotor plane which have been assumed to be well damped and of high frequency in this study, and therefore negligible with respect to the motions of the helicopter. As the gains approach unity the results must be viewed with some apprehension as to their validity. An analysis which includes the blades as a degree of freedom coupled with the helicopter fuselage is needed to determine the exact stability limits. It is probable that these stability limits will not exceed those found from the simpler fixed shaft study, and thus gains above unity will not be investigated.

Comparison of the Complete Feedback Equations (Delta Three and Oehmichen)

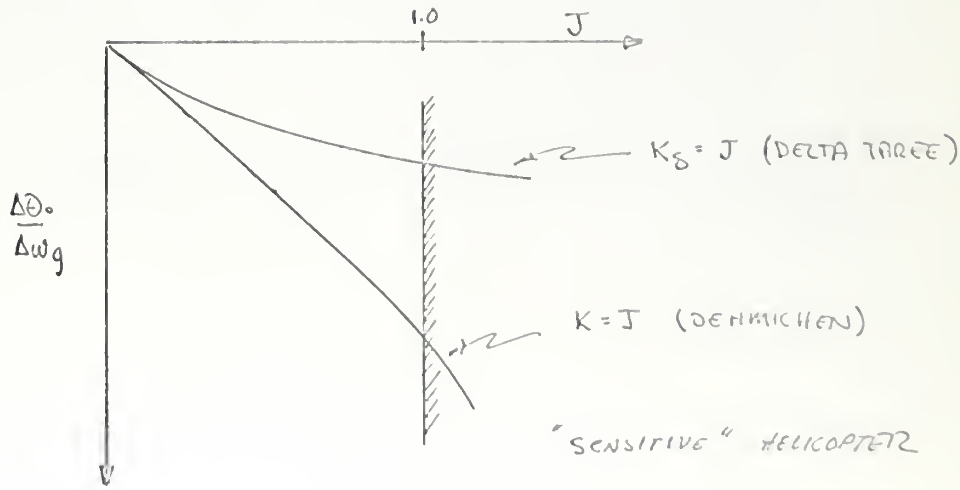
Combinations of the collective and cyclic flapping feedbacks will now be analyzed with emphasis on comparing the effects of the lateral (Delta Three) and longitudinal (Oehmichen) cyclic components. The investigation is in two parts; first, a static analytical analysis which reveals how a particular feedback combination changes the helicopter static stability; and second, analog computer responses showing the effects of the feedbacks on the overall helicopter dynamics.

The signs and magnitudes of the terms in equations II (17) determine the size and magnitude of the left hand side of equations II (16) which can be thought of as modified static stability derivatives, M_w' and Z_w'. Since it is desired that static loads on a helicopter be reduced by means of the feedback system being employed, and since M_{θ_0} and Z_{B_1} are positive, Z_{θ_0} and M_{B_1} are negative, the required signs for equations II (17) are $\Delta B_{1s}/\Delta w_g$ (+) and $\Delta\theta_0/\Delta w_g$ (-). It is evident upon examination of equations II (17) that the two types of feedbacks will not be equally effective for corresponding mechanical gains.

The collective "signals", $\Delta\theta_0/\Delta w_g$, are different due to the differences between $\partial b_1/\partial w$ and $\partial a_1/\partial w$, J" and J_δ". It has been shown that $\partial b_1/\partial w \doteq \partial a_1/\partial w$ at high speeds, and it can be shown from the definitions of J" and J_δ" that J" will always be larger than J_δ" for the same J. In most cases these two values will be relatively close, however, and figure (19b) shows a typical variation of collective static feedback signal with gain, J, for K = -1, 0, +1. The negative values of $\Delta\theta_0/\Delta w_g$ correspond to reducing static normal loads and improving angle of attack stability ($M_{\theta_0}\,\Delta\theta_0/\Delta w_g$). From the figure it is seen that the cyclic component of feedback does not influence the collective feedback signal appreciably. This is because, in equations II (17b), $\partial a_0/\partial w$ is the dominant term, even for large K or K_δ. Among the four data helicopters, however, the 42000 lb one exhibits a marked difference in magnitude between J" and J_δ" at high gains due to a particular configuration of small $\partial a_1/\partial\alpha$ (large K') and large $\partial a_0/\partial\alpha$, $\partial a_1/\partial\theta_0$ resulting in a strong amplification of J" within the range of gains being considered ($-1 < J < +1$). This is shown below.



J" never becomes infinite, but the sensitivity of $\Delta\theta_0/\Delta\omega_g$ (Oehmichen) to the value of gain, J, increases markedly as J increases. In fact, with this helicopter high gain responses could not even be observed on the analog computer due to this high gain sensitivity. A plot of the static feedback signal, $\Delta\theta_0/\Delta\omega_g$ vs J (corresponding to figure (19b), for this "sensitive" helicopter is like the sketch below.



Only near the limits of useable gain, and only for this special configuration does this phenomenon occur, and so, in general, the static collective feedback signal, and thus the initial normal acceleration due to a gust, is relatively independent of the cyclic feedback open loop gain.

In order to evaluate the effectiveness of the static cyclic feedback signal in changing the helicopter static stability, it is helpful to expand all of the terms into the more fundamental rotor aerodynamic derivatives. Before this is done, however, it should be noted that the equations II (17a) are identical in form, of differing sign, and that when J = 0 and K and K_δ are positive, the Oehmichen feedback signal is stabilizing and the Delta Three is destabilizing. As J increases, however, the signals decrease and, for some J, change sign. After much algebra the expanded expressions can be written as:

<u>OEHMICHEN</u>

$$\frac{\Delta B_{1s}}{\Delta \omega_g} = \frac{K'' 2\mu}{\left[1 - \frac{1}{2}\mu^2\right]} \frac{\partial \lambda}{\partial \omega} \left\{ 1 - J' \frac{\gamma}{6} \left(\frac{4}{3} + \frac{\partial \lambda}{\partial \theta_0} \right) \right\}$$

<u>DELTA THREE</u>

$$\frac{\Delta B_{1s}}{\Delta \omega_g} = - \frac{K_\delta'' 2\mu \gamma}{\left[1 + \frac{1}{2}\mu^2\right]} \frac{\partial \lambda}{\partial \omega} \left\{ 1 - J' \frac{\gamma}{2} \left(\frac{\left[1 + \mu^2\right]}{4} + \frac{1}{3} \frac{\partial \lambda}{\partial \theta_0} \right) \right\}$$

The static cyclic feedback signals will thus change sign when

<u>OEHMICHEN</u>

$$J' > \frac{6}{\gamma} \frac{1}{\left[4/3 + \partial\lambda/\partial\theta_0 \right]}$$

<u>DELTA THREE</u>

$$J' > \frac{8}{\gamma} \frac{1}{\left[(1+\mu^2) + \frac{4}{3}\frac{\partial\lambda}{\partial\theta_0} \right]}$$

In the absence of induced effects, as a high speed approximation, and for $\gamma \doteq 12$ the feedbacks change sign when

<u>OEHMICHEN</u>

$$J' \doteq\!> 3/8$$

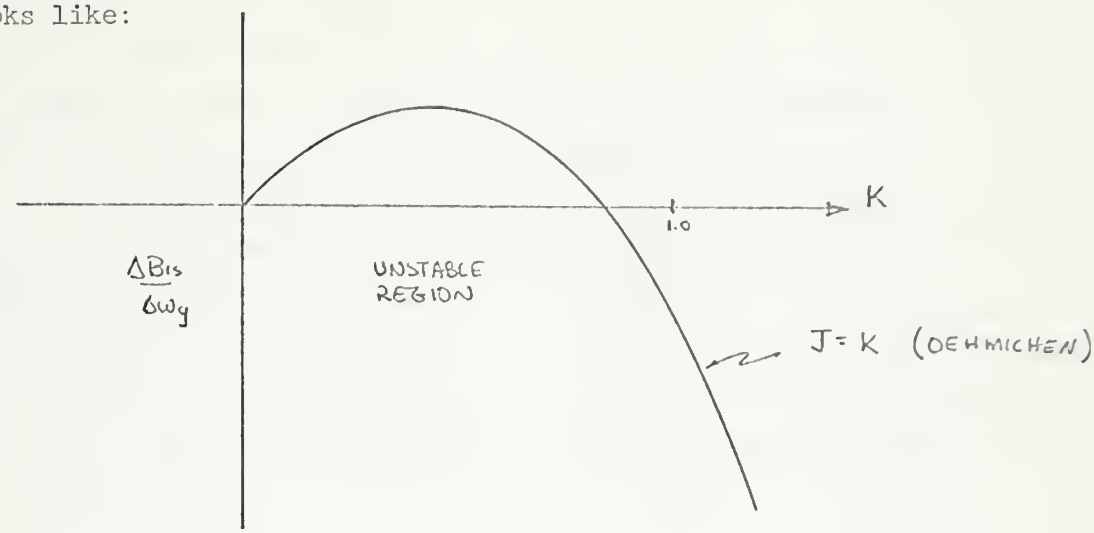
<u>DELTA THREE</u>

$$J' \doteq\!> 2/3$$

It was shown before that J' is gain limited at about 2/3, but for $J<1$ (blade flapping stability limit), J' never will exceed about 1/2, even at the highest speeds. With this information, it is predictable that the Delta Three cyclic feedback component will always be statically destabilizing, and that at high gains and high forward velocities the Oehmichen cyclic feedback component will change from stabilizing to destabilizing. This prediction is fairly independent of anything but airspeed. As the airspeed is reduced the induced effects boost the destabilizing limit of J', and, at the same time, reduce the actual value of J' so that at low speed there is no change in sign of the feedback.

The prediction is verified in figure (19a) where the static cyclic feedback signal is plotted for the Oehmichen and Delta Three feedbacks. It is seen that, for the conventional mechanical arrangement of Oehmichen pitch flap coupling, the static feedback signal changes to the "wrong" sign as the mechanical gain is increased to near unity on the 24000 lb helicopter. The same result is found for all four data helicopters, the crossover point being close to that shown in figure (19), but the slope of the curve varying with configuration. The same configuration (42000 lb helicopter) that showed a marked collective feedback sensitivity also shows a marked cyclic feedback sensitivity, or increased [illegible] slope of the curve in figure (19a).

It looks like:



The reason for this is the increased importance, in the cyclic feedback, of the increasingly larger collective feedback signals, as the mechanical gain of the system is increased.

As a summary of the findings of this static analysis, it can be simply said that; 1) Positive collective feedback reduces static loads (normal as well as angular) almost independently of the sign or magnitude of the cyclic feedback, and M_{θ_0} plays an important part in how "good" this type of feedback is.

2) Positive Delta Three cyclic component always increases the static angle of attack instability of the helicopter, and negative Delta Three is always stabilizing.

3) Oehmichen cyclic component will improve static stability for most of the range of useable gain, J, but, at some high gain and high forward velocity, will begin to vitiate the static stability.

From these static results arises a very interesting helicopter stabilization technique - negative Delta Three hinges on the main rotor. It has been shown that this technique improves static stability, but two questions must be answered before it can be seriously considered. First is the question of blade motion stability (this work cannot uncover blade instability because of the assumptions made about the rotor plane) which immediately comes up because of the nature of the pitch-flap coupling (the pitch of a blade is increased when it flaps up) is intrinsically unstable. This matter has been investigated in reference (6) where it is shown that "The use of negative Delta Three introduces the possibility of flapping divergence but this is not a problem for normally required values of negative Delta Three".

The second question concerns the increase in Zw' associated with negative Delta Three hinges. According to the analysis of this work, in which only lowest order rotor plane dynamics are considered, one effect of negative Delta Three is to increase static normal gust loads. This is undesirable, but the validity of neglecting any higher order dynamics (gust alleviation as defined in Appendix IV) is questionable, especially with feedbacks such as these, and it is doubtful if the adverse effects of negative Delta Three are as pronounced as they seem in this analysis. A more complete analysis, however, is needed to verify the above assertion.

The dynamic responses of one of the data helicopters with various feedback component combinations are presented as figures (20 - 23). As expected, those feedback combinations which improve static stability (M_α) are also the ones which improve the dynamic gust responses. As such, negative Delta Three (figures 21 and 23b) feedback is clearly the most promising feedback of the various ones investigated.

Figures (20 and 21) show that gust responses are improved for:

OEHMICHEN	DELTA THREE
K = constant, J increasing	J = constant, K decreasing
J = constant, (K = some intermediate value before crossover to negative feedback)	(increasing negatively)

The analog computer responses are revealing and, in the case of the negative Delta Three feedbacks, the improvements in gust response are seen to be excellent.

All of the results above are in agreement with, and may explain, the surprisingly successful flight test results briefly reported in reference (6). In that test a helicopter was fitted with negative Delta Three hinges ($\delta_3 = -39^{\circ}$, $K_{\delta} = .81$) and flown to its power limit. The investigation was directed at rotor blade flapping stability and amplitude, and so no data on the stability of the helicopter is presented in that report.

Oehmichen and Delta Three Variations; Rate and Attitude

Several helicopter feedback concepts that have been introduced in the literature can be quite adequately described by variations of the Oehmichen and Delta Three feedback components when only the macroscopic dynamics of the helicopter are being investigated. In reference (9) is proposed a thrust and tilting moment feedback system, in reference (13) is presented the Lockheed gyro system, and in reference (4) are presented various stabilization schemes. All of the above fit into this category.

The proposed thrust feedback can be pretty well described by the integral of the collective component of Oehmichen feedback, $\Delta\theta_0 = -J\int \Delta a_0 \, dt$, where it is an adequate approximation that $\Delta a_0 \propto \Delta T$.

From experience with the non-integrated feedback ($\Delta\theta_o = -J\,\Delta a_o$) it is predictable that M_{θ_o} plays an important part in the effectiveness of the integrated feedback. As can be seen from figure (24), when M_{θ_o} is zero this feedback scheme vitiates the helicopter response, but when M_{θ_o} is positive, increasing the gain increases the frequency and decreases the time to double amplitude of the response. This is similar to the result found with the non-integrated collective feedback where the static angle of attack instability was decreased due to the cross control derivative.

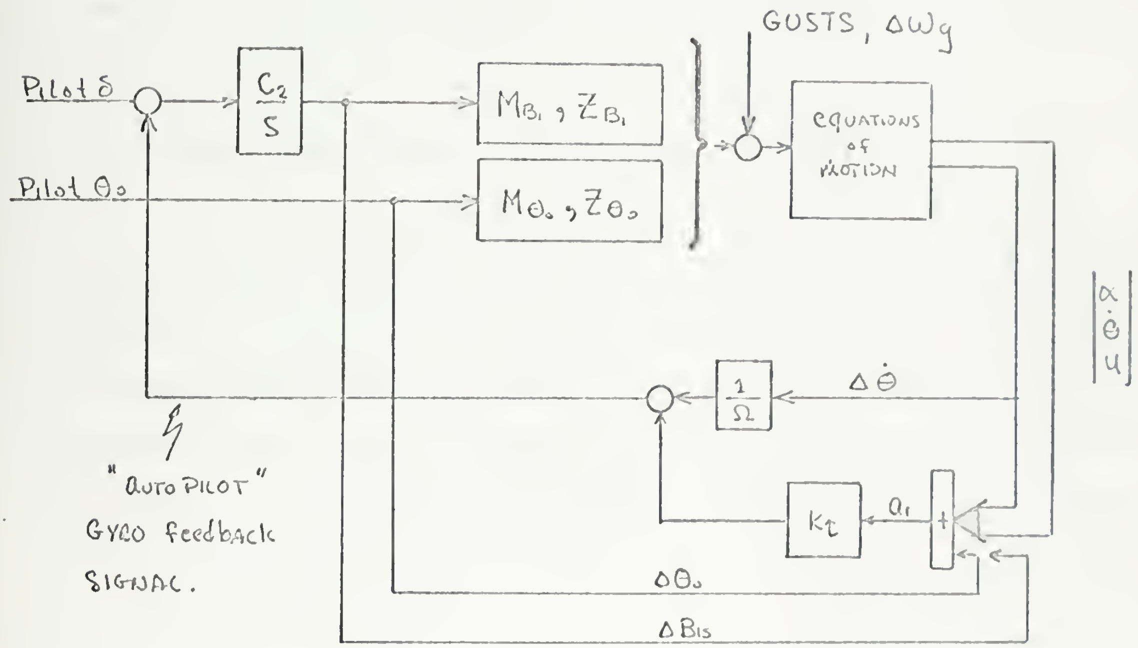
The integral coning angle feedback (thrust feedback) does not reduce static normal loads, and from figure (24) evidently reduces the effective Zw of the helicopter out of phase with the angle of attack variation (the integration makes the loop too "slow"), and thus only improves helicopter dynamic response if the cross control derivative, M_{θ_o} is large and positive.

Figure (25a) shows the results of integrating the cyclic component of Oehmichen feedback, $B_{1S} = K\int \Delta a_1\, dt$ as proposed in reference (4). No static stability derivatives are modified with this scheme and helicopter dynamic gust responses are not improved. Again, the integration makes the feedback too "slow".

Combining the two above, or integrating the entire Oehmichen feedback signal, results in the gust responses of figure (25b). As with the individual component feedback signals, the loop is too "slow" and does not improve the response of the helicopter. This particular combination is the same as that proposed in reference (9), the Thrust and Tilting Moment feedback.

The one variation of the Oehmichen feedback that deserves special attention at this time is the Lockheed gyro system. This system has been devised in order to relieve dynamic rotor loads and to stabilize the helicopter, as a whole, in the presence of gusts. The first feature cannot be treated within the scope of this work, and so the model used for the system herein idealizes the workings of this feature, and only investigates the second alleged feature.

In the ideal system $L = 0, V = 1$ (in the nomenclature of reference 13) and the feedback reduces to the integral of Oehmichen plus the integral of pitching rate, for the purposes of this work. A block diagram of the idealized system is shown below.



It is not possible to perform the same static feedback analysis as was done on the Oehmichen and Delta Three systems because of the integration which does not "let through" any static feedback. A feedback equation can be written, however, which resembles the ones of the previous sections. From the diagram:

$$\Delta B_{1s} = \frac{C_2}{\left[S + K_\tau \frac{\partial a_1}{\partial \alpha} C_2 \right]} \left\{ K_\tau \left(\frac{\partial a_1}{\partial \alpha} \Delta \alpha + \frac{\partial a_1}{\partial \dot{\theta}} \Delta \dot{\theta} + \frac{\partial a_1}{\partial u} \Delta u \right) + \frac{\Delta \dot{\theta}_f}{\Omega} \right\}$$

where K_{τ} = "sweep constant" = $\frac{F(p^2-1)}{2}$ in reference (13)

and C_2 = linkage between gyro and swash

Except for the added pitching rate term, and the lag time constant associated with the feedback (s in the denominator), this Gyro feedback equation resembles the Oehmichen equation. Typical values for K_{τ} are $0 \rightarrow 1/2$ and $C_2 = 2/3$.

It was found before that $\partial a_1 / \partial \alpha = 2\mu^2 / \left[1 - \frac{1}{2}\mu^2\right]$

so that even with large K_{τ} there is a long lag time, or the feedback loop is "slow". Because of this long lag time the gyro feedback does not appreciably alter the motions before the angle of attack convergence, and hence the normal acceleration response, is completed. Since there is no collective feedback there is only a small alteration in the normal force equation ($Z_{B_1} \Delta B_1$). The lagged rate feedback acts as a torsional spring, but slows the rotational motions only slightly. For all of these reasons the analog computer responses (figure 25c) for the idealized Lockheed gyro feedback system are sorely disappointing. The responses are for $C_2 = 2/3$ and increasing K_{τ}.

As a standard for comparison of all of these feedback techniques that have been looked at, it is now appropriate to show the gust response of the helicopter equipped with conventional SAS. Figure (26) presents the analog computer responses of two of the data helicopters modified by rate and attitude feedback - the last two strips of the figure are for values of τ_1, τ_2 and R with which the production models of those helicopters are presently equipped.

The helicopter responses are, at first glance, quite "good". It seems that there should be little reason for a pilot to complain. Because they do, however, it becomes necessary to look more closely at the responses and at the model used for obtaining these responses. Only the [illegible] will be touched

on here, as this work is not designed to provide very accurate detailed models.

In the figure it is seen that the acceleration responses, while heavily damped, exhibit considerable static (time = 0^+) values and over shoot. Because this feedback does not alter the static response (M), gust induced loads (normal and torque about the center of gravity) are unchanged. This means that the pilot will still feel the same local "g" (i.e. the normal load, $Z_w w_g$, plus the upward angular acceleration times his distance from the C.G.) response as he did without the SAS. Even though the motions are then dissipated quickly, the pilot is still subject to the same maximum forces on his body with this feedback.

It is evident, from the point of view of this study, that the improvement in angular static stability is the prime determinant of the quality of a feedback technique. As such, no flapping feedback system with an integrator in the loop, and no body mounted SAS which does not sense angular acceleration will appreciably improve the helicopter pilot's opinion of the flying qualities of his craft.

CONCLUSIONS

Parts I and II, being somewhat separate from each other in material presented, warrant separate conclusions. A summary of the findings in Part I is given on pages (28 and 29) as the physical interpretation of the similarities and differences found in that part. In addition:

1) Airplane and helicopter Z_w's are found to be quite nearly equal when comparing aircraft of similar high speed (150 kts) design conditions. This leads to the discovery that the normal acceleration gust response of the two types of aircraft is similar.

2) The size of $M_{\dot{\theta}}$ and the sign and magnitude of M_{α} are different for the two types of aircraft, the helicopter being on the unstable side in both cases. This leads to the realization that the rotational motions are of prime importance in helicopter gust response.

3) The influence of the fuselage, tail, and rotor flapping hinge offset (or non-articulated blades) becomes more important than, and in many cases overshadows, "conventional" helicopter stability characteristics in high speed flight with the "typical" helicopter characterized herein. This is to say that rotor flapping becomes less important in the presence of these other factors.

4) Of the various stabilization techniques involving rotor flapping as the basis for the feedback signal studied in this work, negative Delta Three (positive pitch-flap coupling) showed the best (and only acceptable) results. The static stability is improved considerably with this type of feedback. It is considered that this finding warrants further detailed study in this area.

5) Integrated feedbacks, the signals for which originate as rotor flapping variables, are too "slow" to improve the gust response characteristics of the helicopter. Because of the integrator in the loop, there is no change in static stability and consequently no improvement in the initial gust response.

REFERENCES

1. Amer, K. B.: Theory of Helicopter Damping in Pitch or Roll and a Comparison With Flight Measurements. NACA TN-2136, October 1950.

2. Curtiss, H. C., Jr.: Lecture Notes from Course AMS 567-568, "V/STOL Performance, Stability, and Control". Princeton University, 1968-69.

3. Curtiss, H. C., Jr.: Umpublished Work on Blade Dynamics and Gust Response of Helicopters.

4. Daughaday, H. et. al.: Helicopter Handling Qualities Investigation, Phase II, Part C. Cornell Aeronautical Laboratory, June 1955.

5. Drees, J. M., and Harvey, K. W.: Helicopter Gust Response at High Forward Speed. AIAA Paper 68-981, October 1968.

6. Gaffey, T. M.: The Effect of Positive Pitch-Flap Coupling (Negative 3) on Rotor Blade Motion Stability and Flapping. American Helicopter Society Paper 227, May 1968.

7. Gessow, A. and Myers, G. C.: Aerodynamics of the Helicopter. F. Ungar Publishing Co., New York. 1967.

8. Gould, D. G.: Note on the Gust Alleviation Factor for Helicopters and Autogyros. National Research Council of Canada, Aeronautical Report LR-464, September 1966.

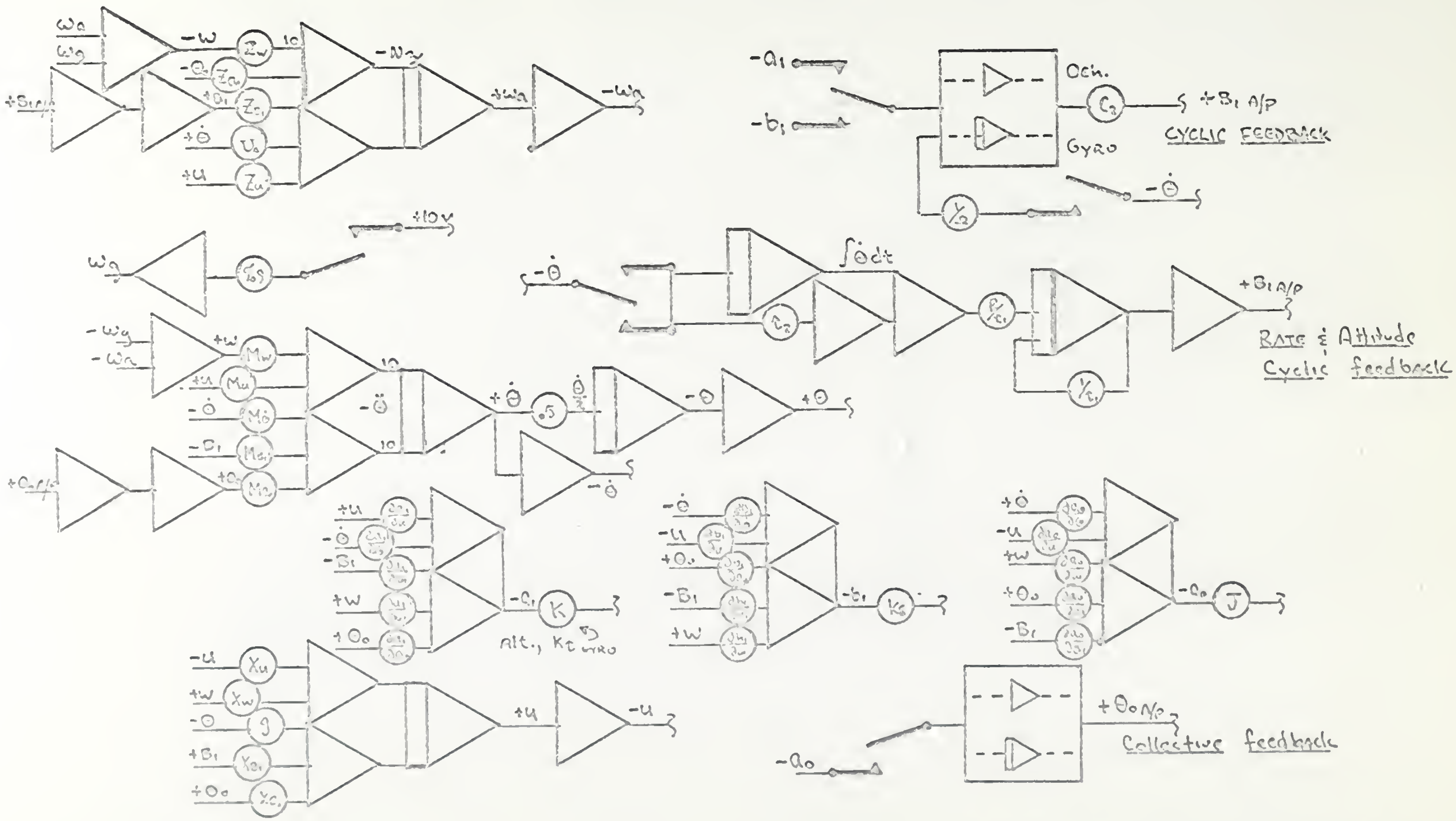
9. Hohenemser, K., and Johnson, R. L.: Dynamics of Lifting Rotors With Thrust and Tilting Moment Feedback Controls. American Helicopter Society Paper, May 1969.

10. McRuer, D., Ashkenas, I., Graham, D.: Aircraft Dynamics and Automatic Control. Systems Technology Inc. and Naval Air Systems Command, August 1968.

11. Seckel, E.: Stability and Control of Airplanes and Helicopters. Academic Press, New York. 1964.

12. Seckel, E., and Curtiss, H. C., Jr.: Aerodynamic Characteristics of Helicopter Rotors. Princeton University, Dept. of Aerospace and Mechanical Sciences Report No. 659, December 1963.

13. Sissingh, G. J.: Response Characteristics of the Gyro-Controlled Lockheed Rotor System, Lockheed-California Report 131, May 1967. (Presented at American Helicopter Society, 23rd Annual Forum, May 1967.



All VARIABLES NORMALIZED TO MAX VALUES.

FIGURE (1): Analog Computer Schematic

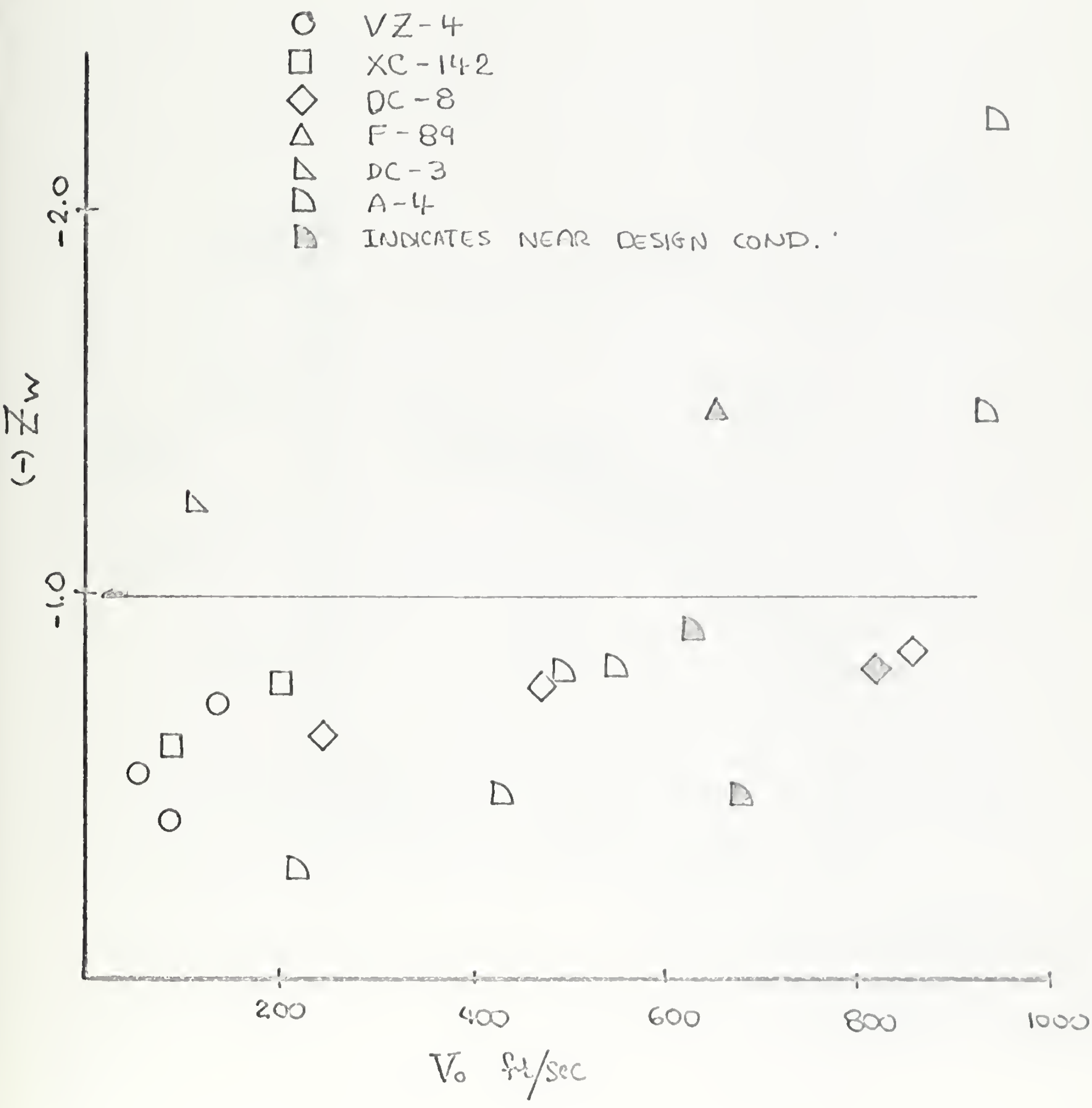


FIGURE (2): Zw v. V_0 for various airplanes.

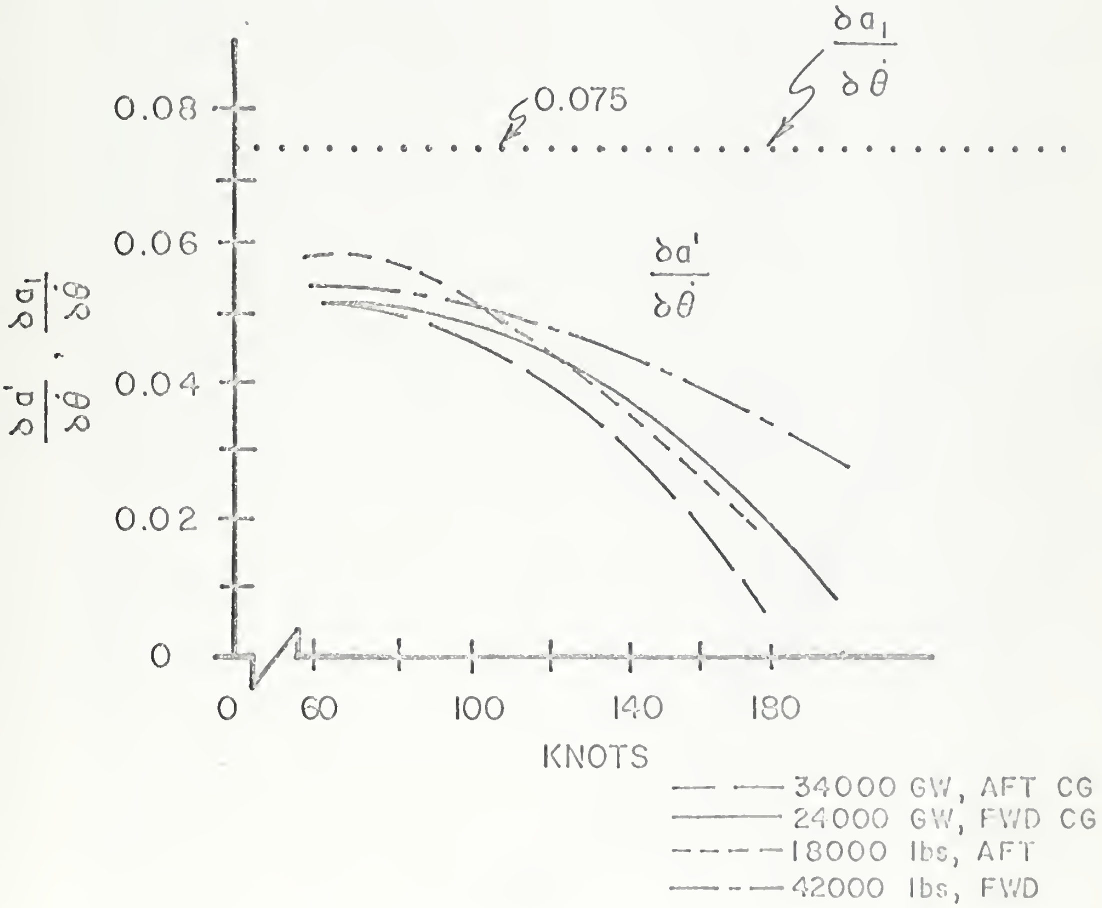


FIGURE (3): $\frac{\partial a'}{\partial \dot\theta}$ and $\frac{\partial a_1}{\partial \dot\theta}$ vs. U_o

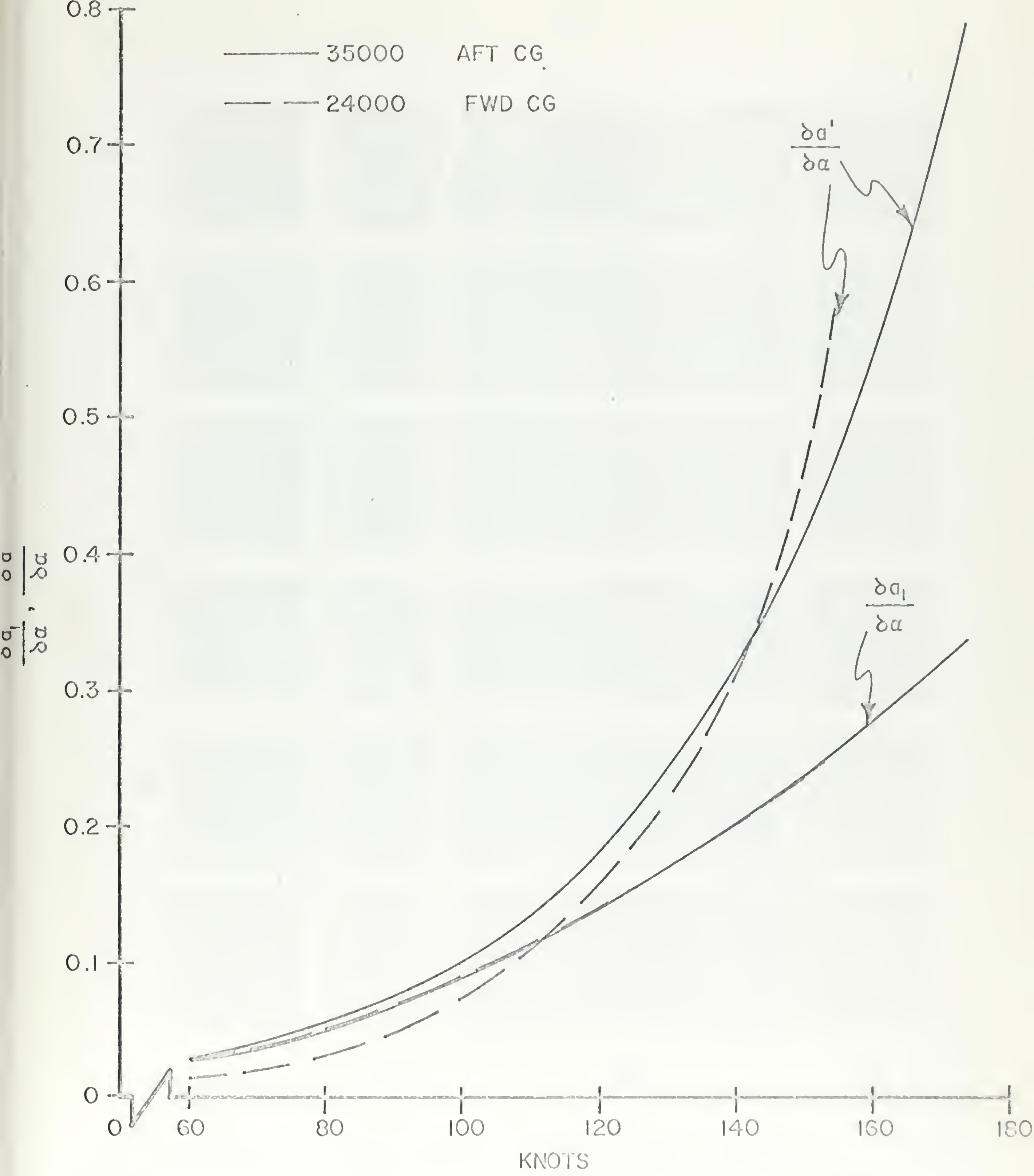


FIGURE (4): $\frac{\partial a_1}{\partial \alpha}$ and $\frac{\partial a'}{\partial \alpha}$ vs. U_o

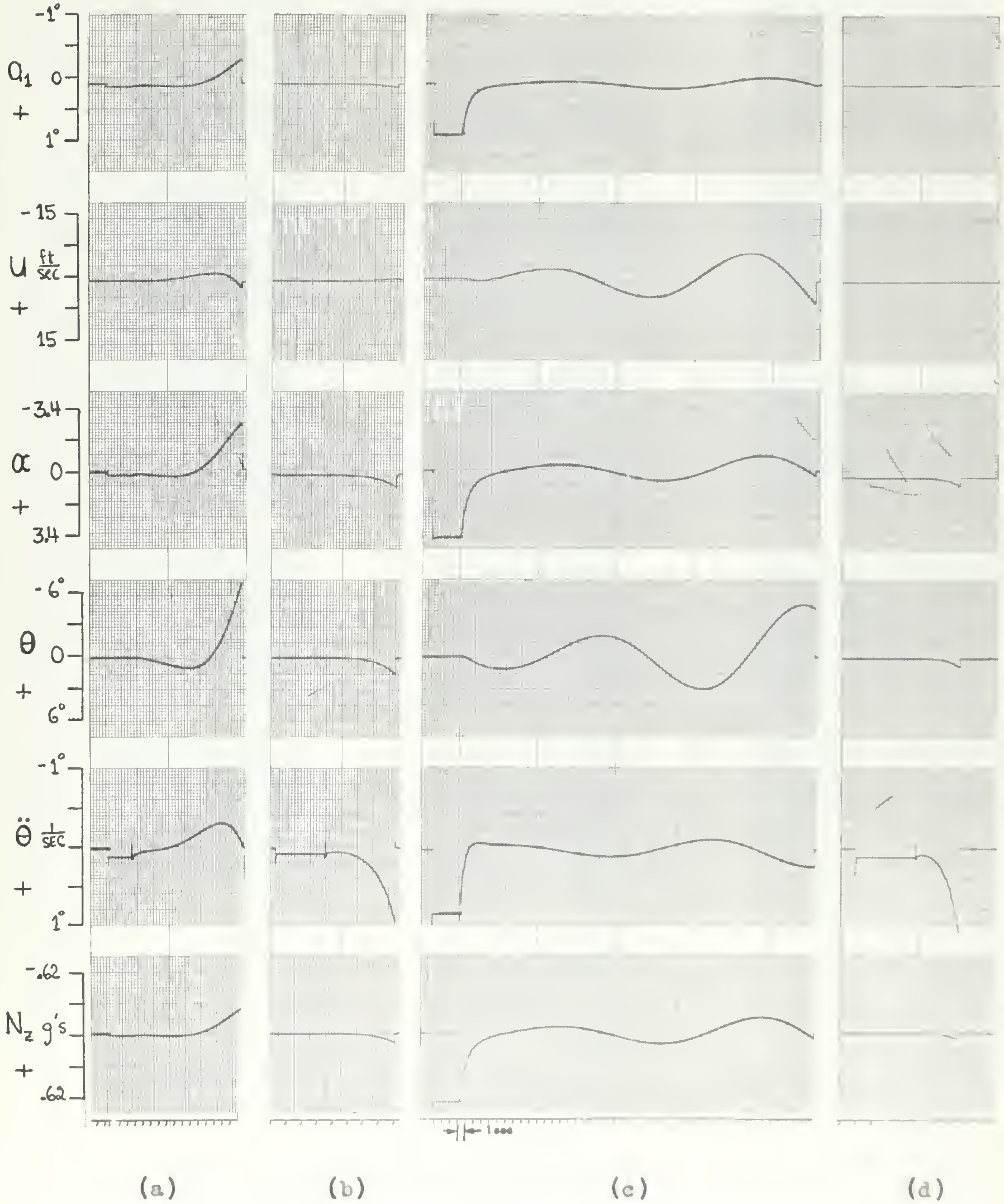


FIGURE (5). Three degrees of freedom analog responses for basic helicopters. a) 42000 lb, b) 35000 lb, c) 24000 lb, d) 18000 lb.

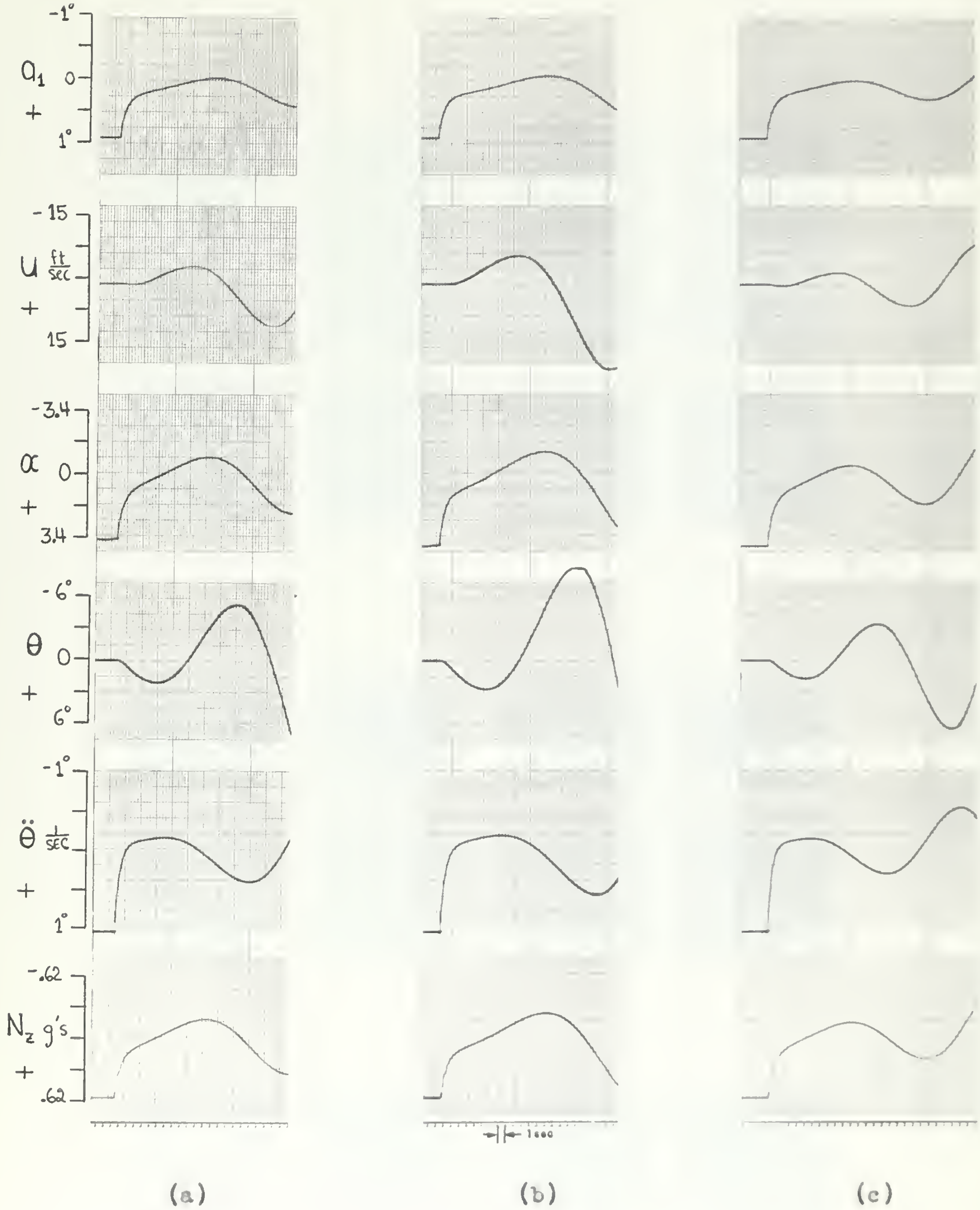
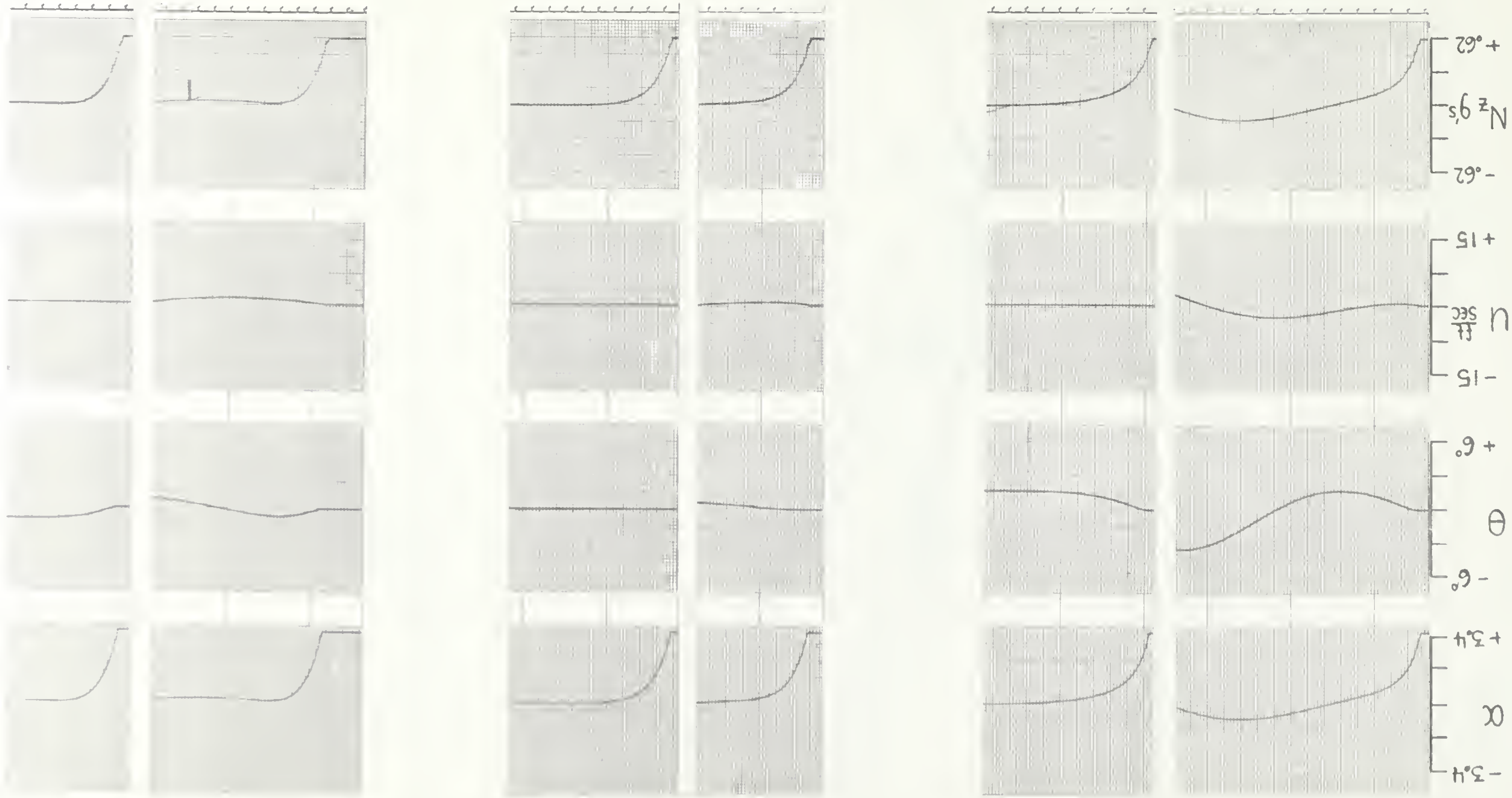


FIGURE (6): Analog responses for "average" helicopter (except Zw and Mw), a) derivatives at "average" value, b) lowest values, c) highest values.



(a) (b) (c)

FIGURE (7): Three degrees and two degrees of freedom analog traces compared, a) Mw positive, b) Mw=0, c) Mw negative.

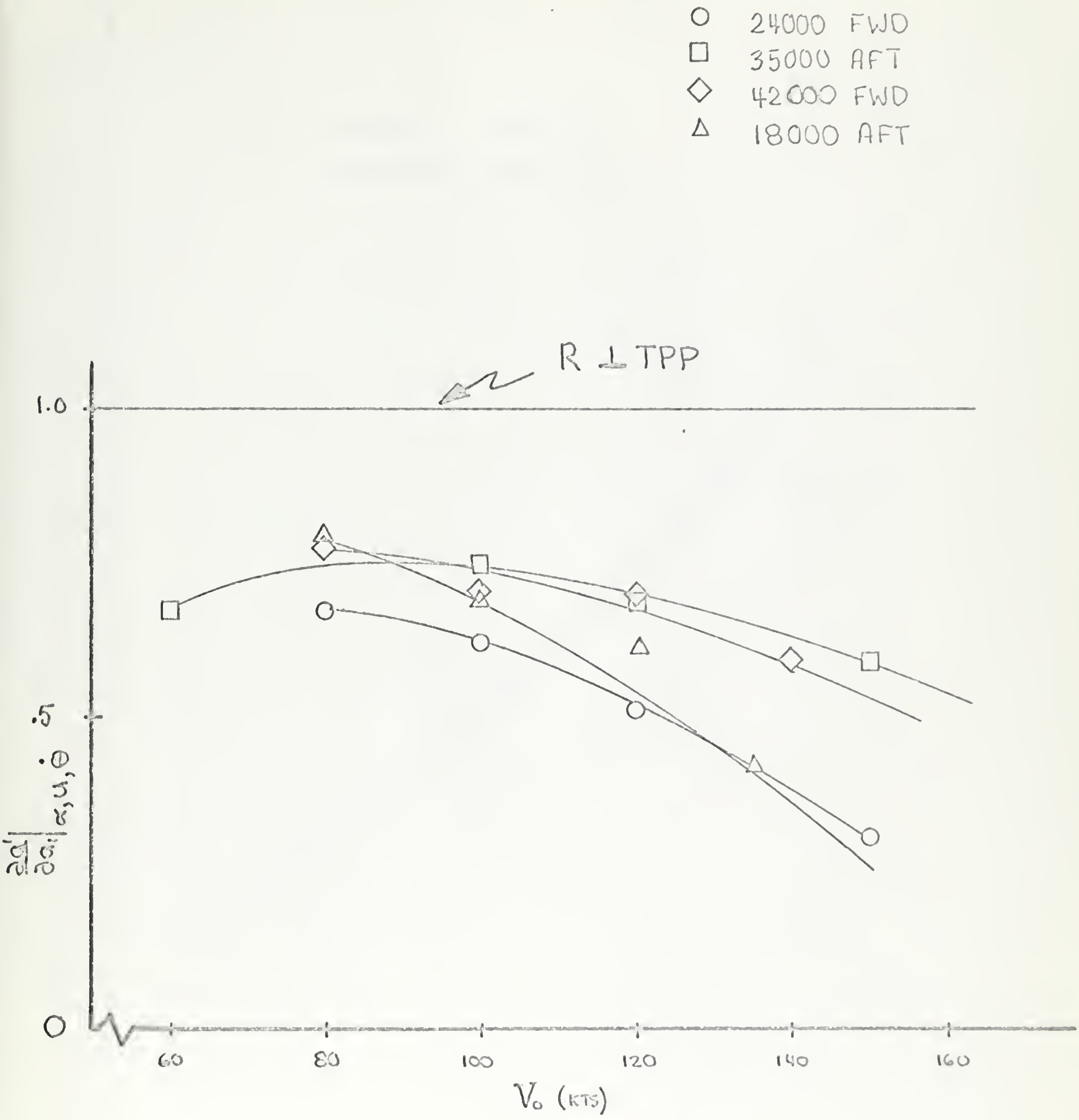


FIGURE (8): $\frac{\partial a'}{\partial a_1}$ vs. V_0 for data helicopter.

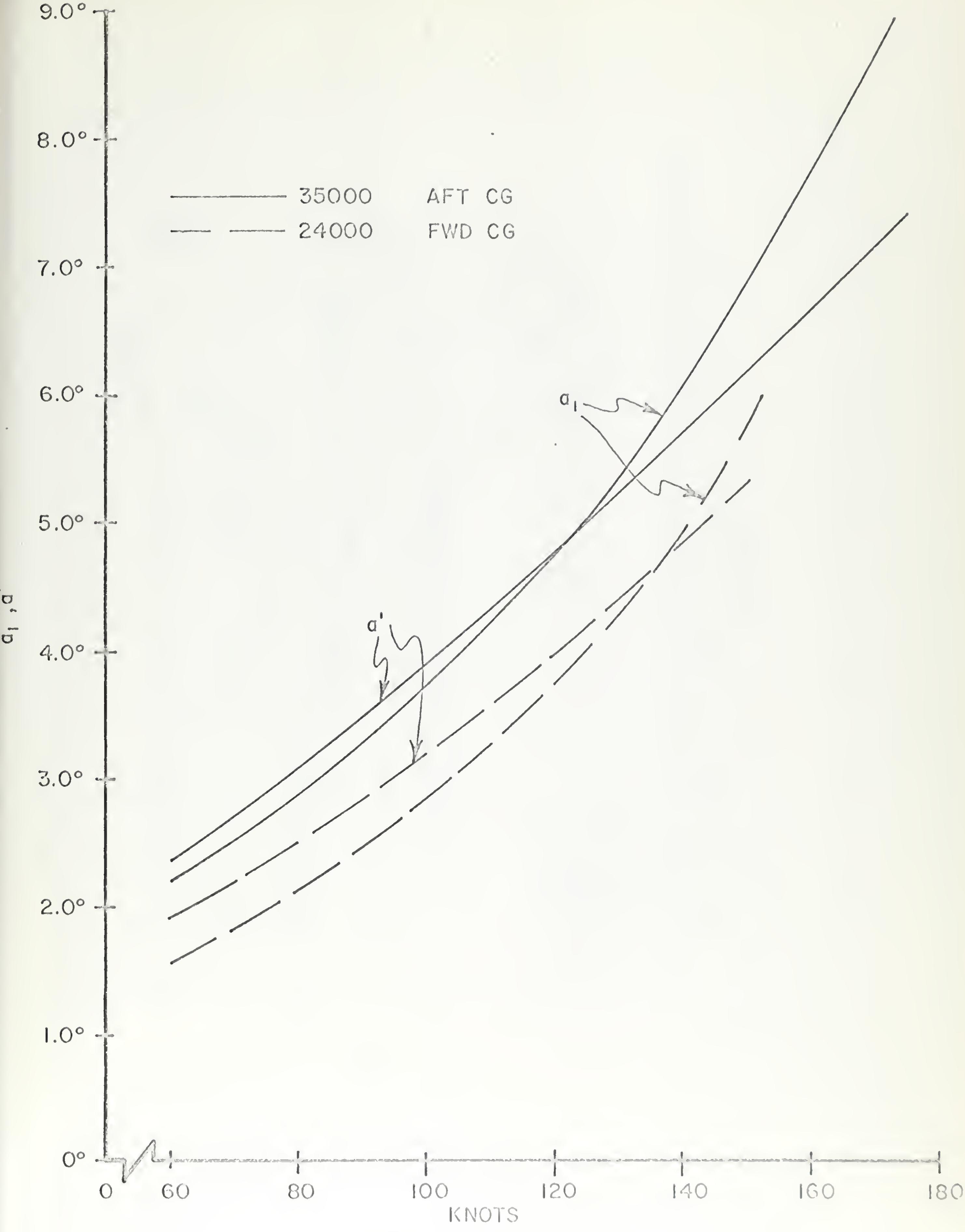


FIGURE (9): a_1 and a' vs. U_o

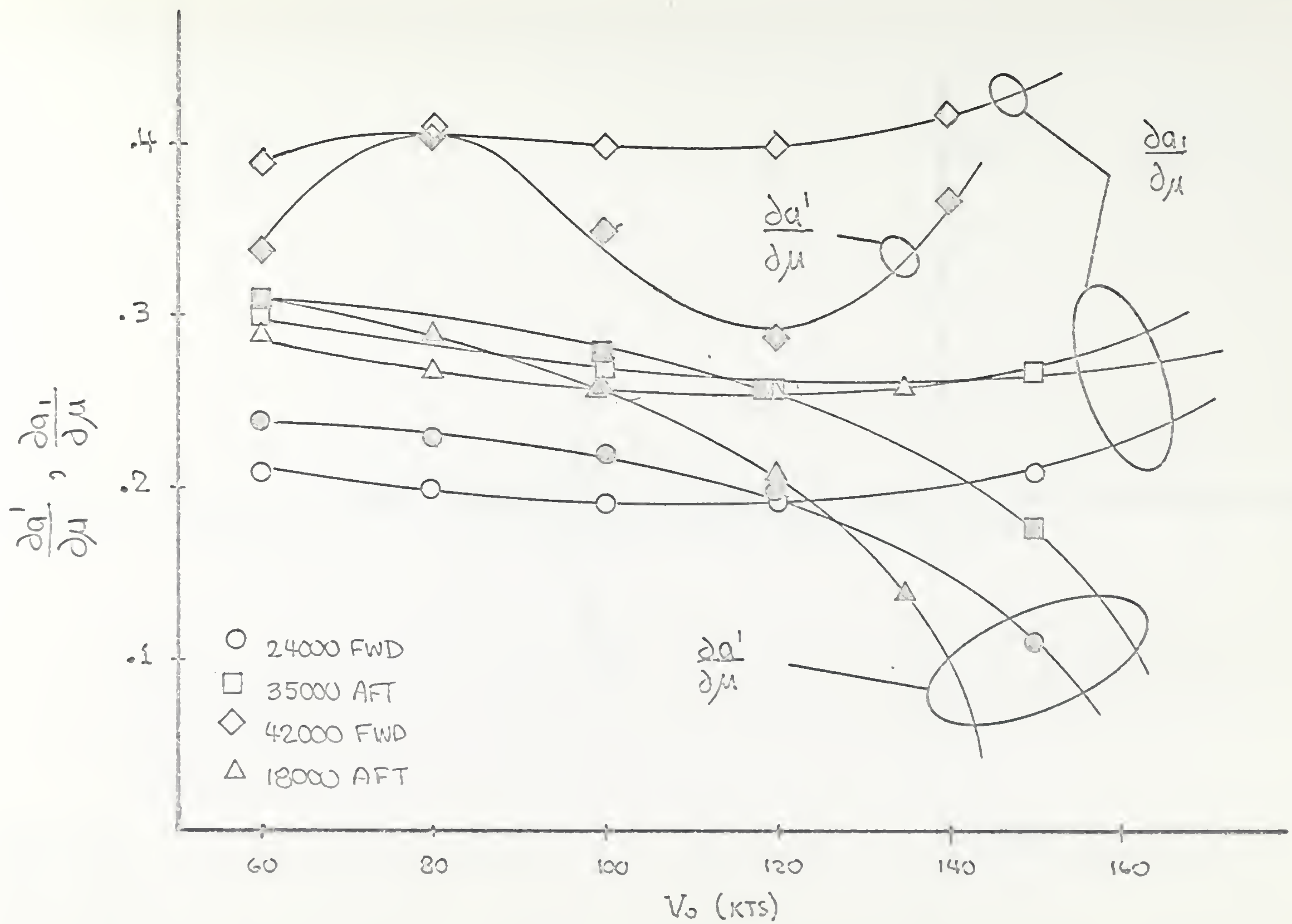


FIGURE (10): $\frac{\partial a'}{\partial \mu}$ and $\frac{\partial a_1}{\partial \mu}$ v. V_o for data helicopters.

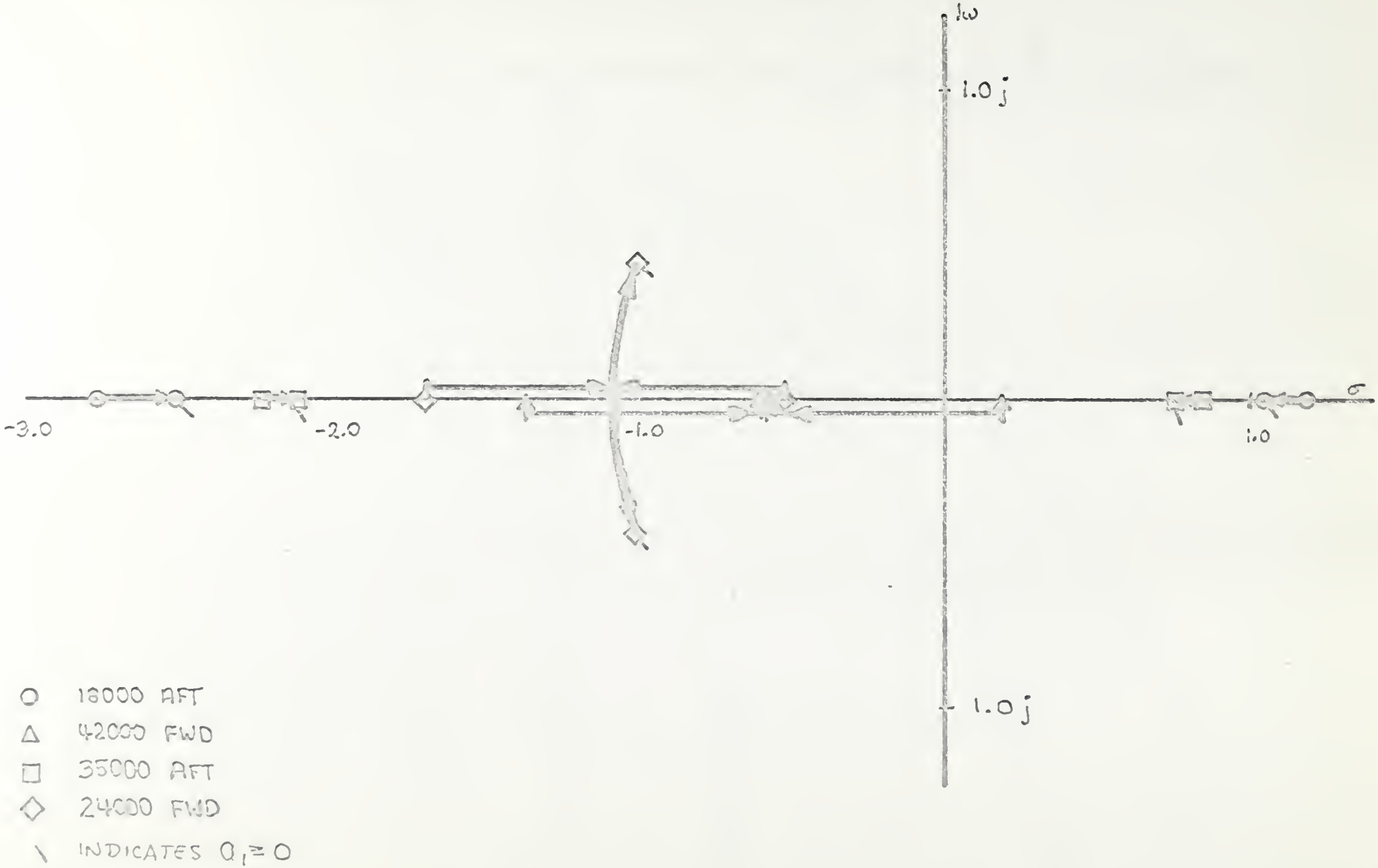


FIGURE (11): Characteristic roots of data helicopters with and without longitudinal flapping.

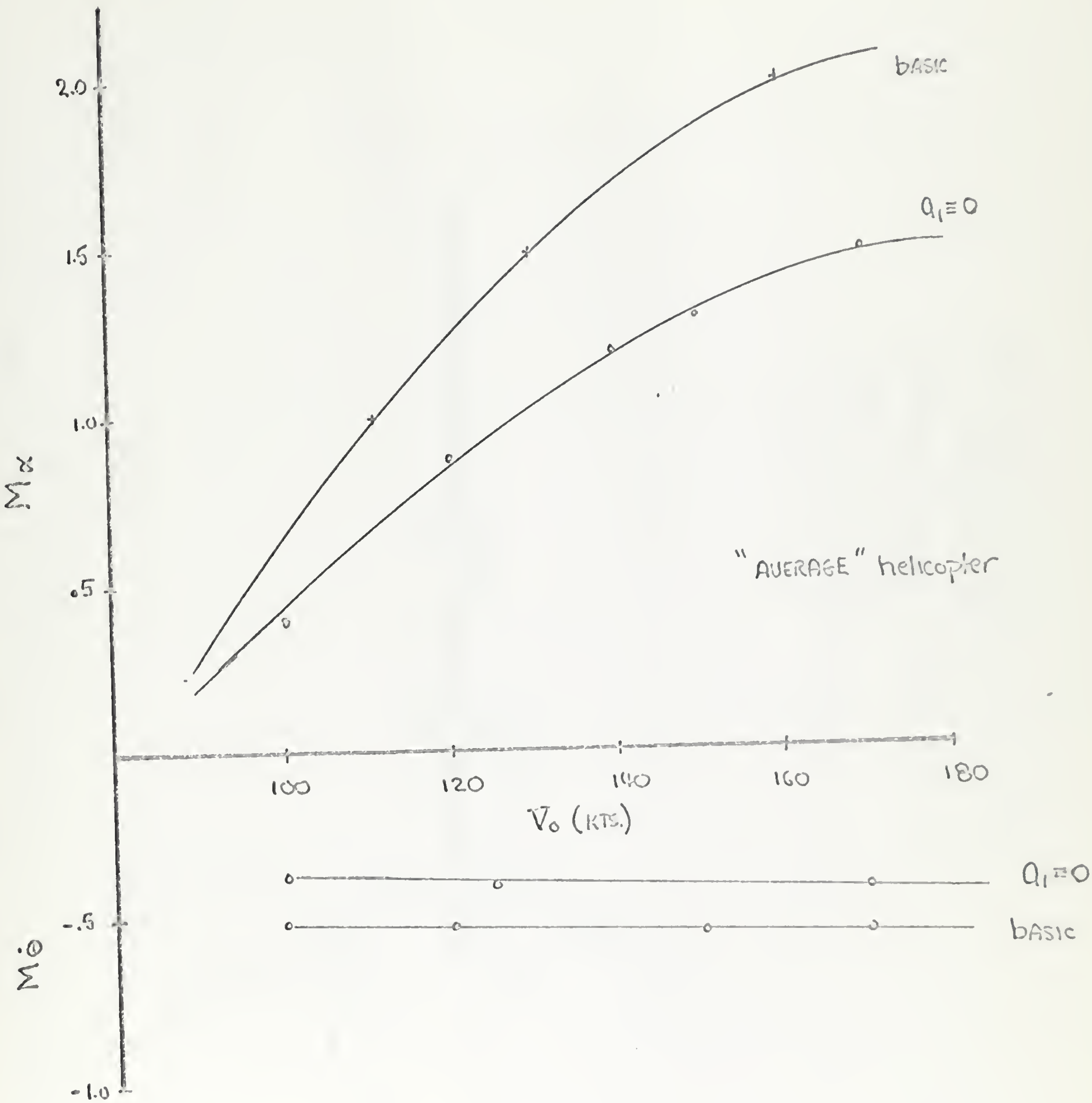


FIGURE (12): M_α and $M_{\dot\theta}$ compared for $a_1 \equiv 0$ and $a_1 \not\equiv 0$.

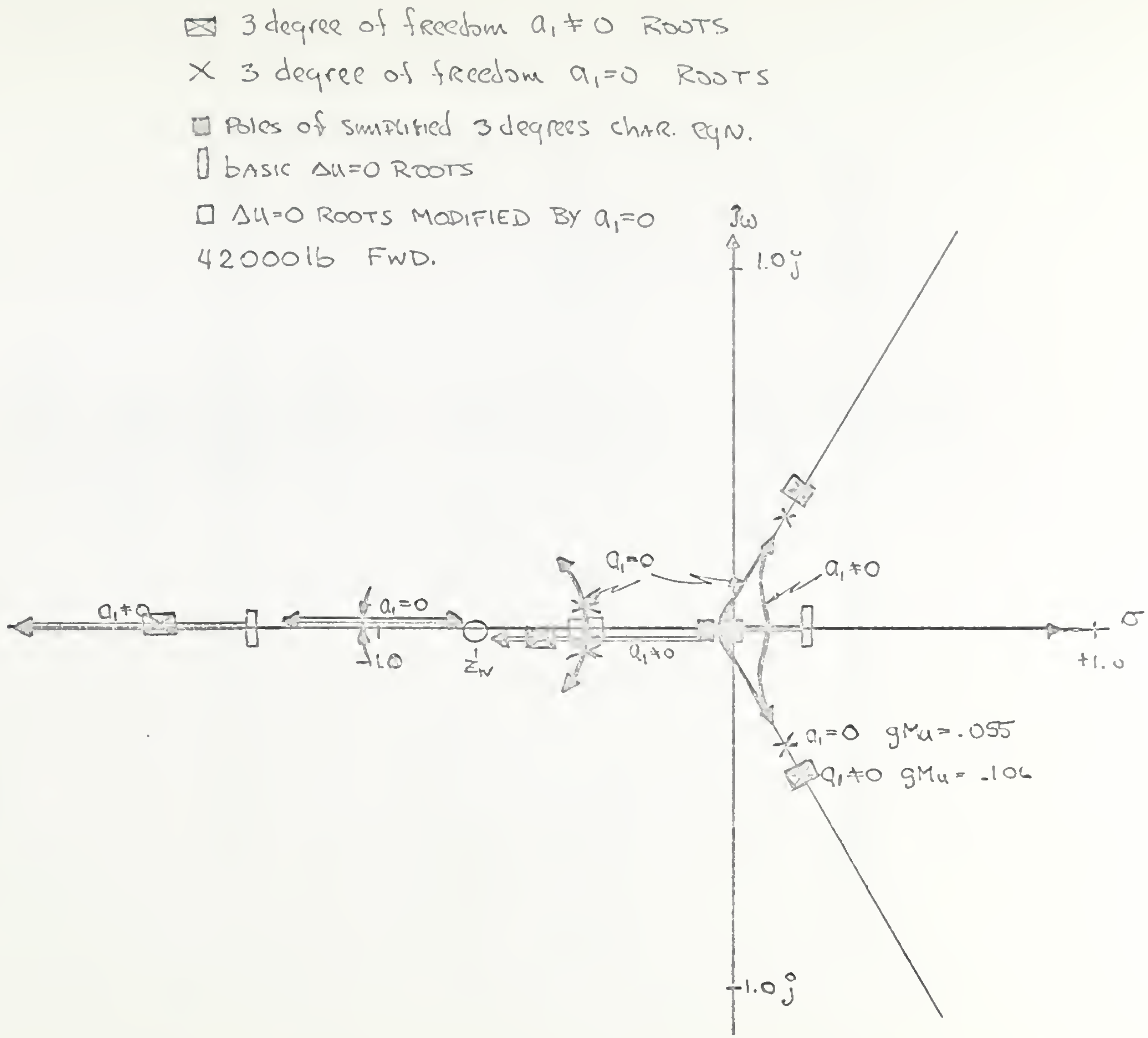


FIGURE (13): Root Loci for positive Mu; $a_1 \neq 0$, $a_1 = 0$. Simplified characteristic equation $s^2(\Delta u = \text{roots}) + g\,Mu(s - Zw) = 0$.

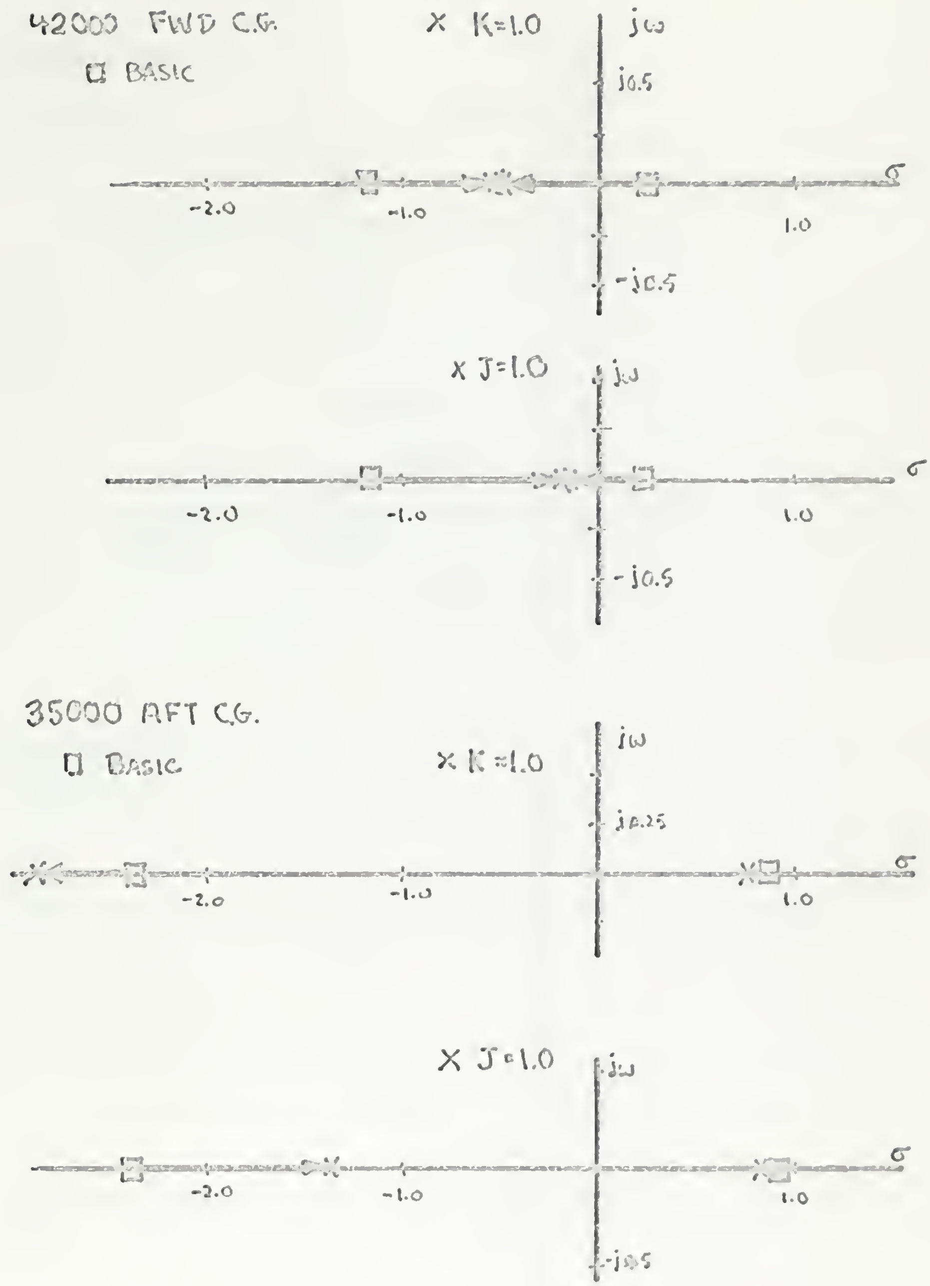


FIGURE (14): Characteristic roots for each data helicopter with longitudinal (top), and coning (bottom) feedbacks.

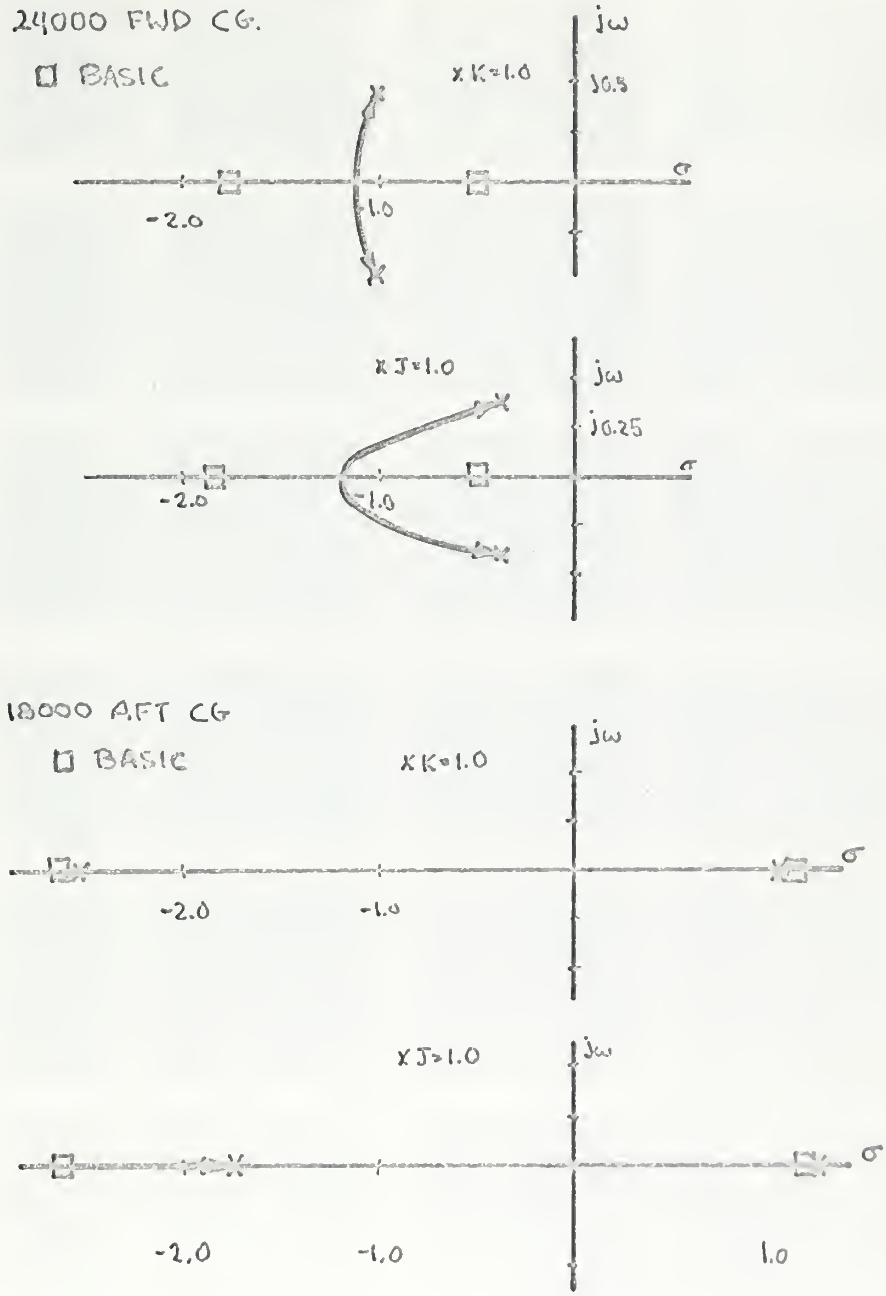


FIGURE (14): Continued.

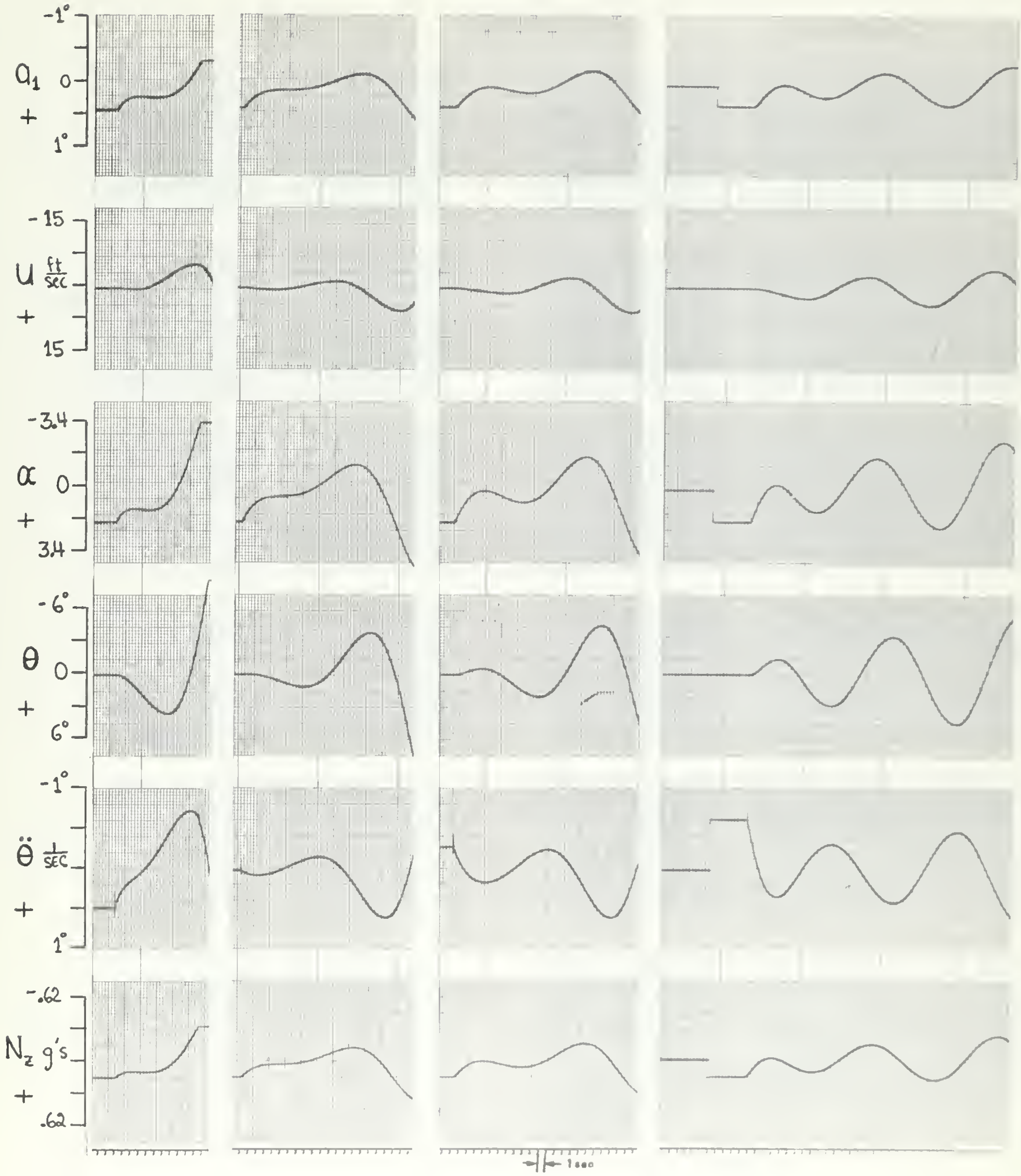


FIGURE (15): Longitudinal (Oehmichen) flapping feedback, $B_1 = K a_1$, for $Z_{B_1} = 0$ K = .25, 0.5, .75, 1.0 (42000 lb helicopter).

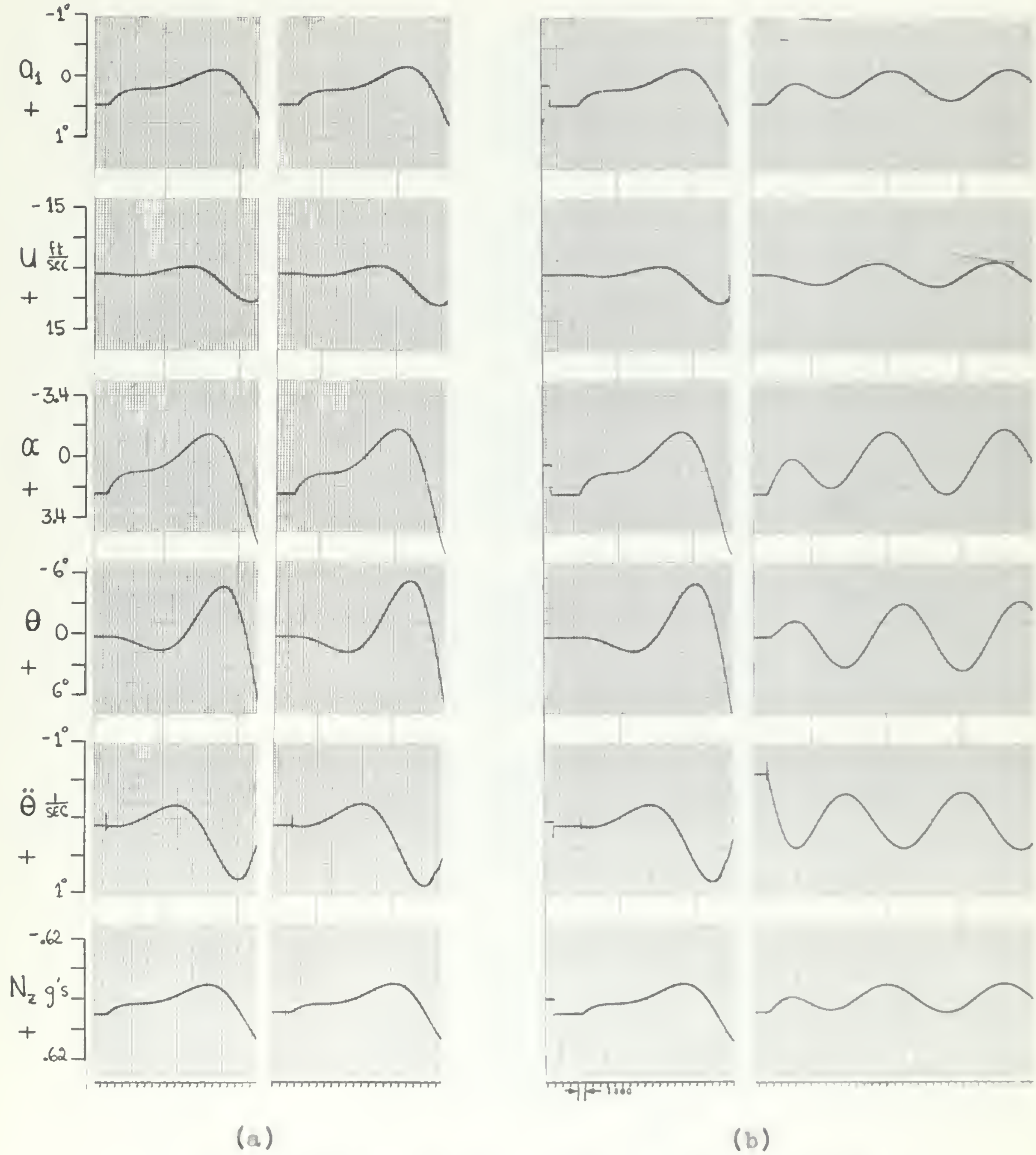
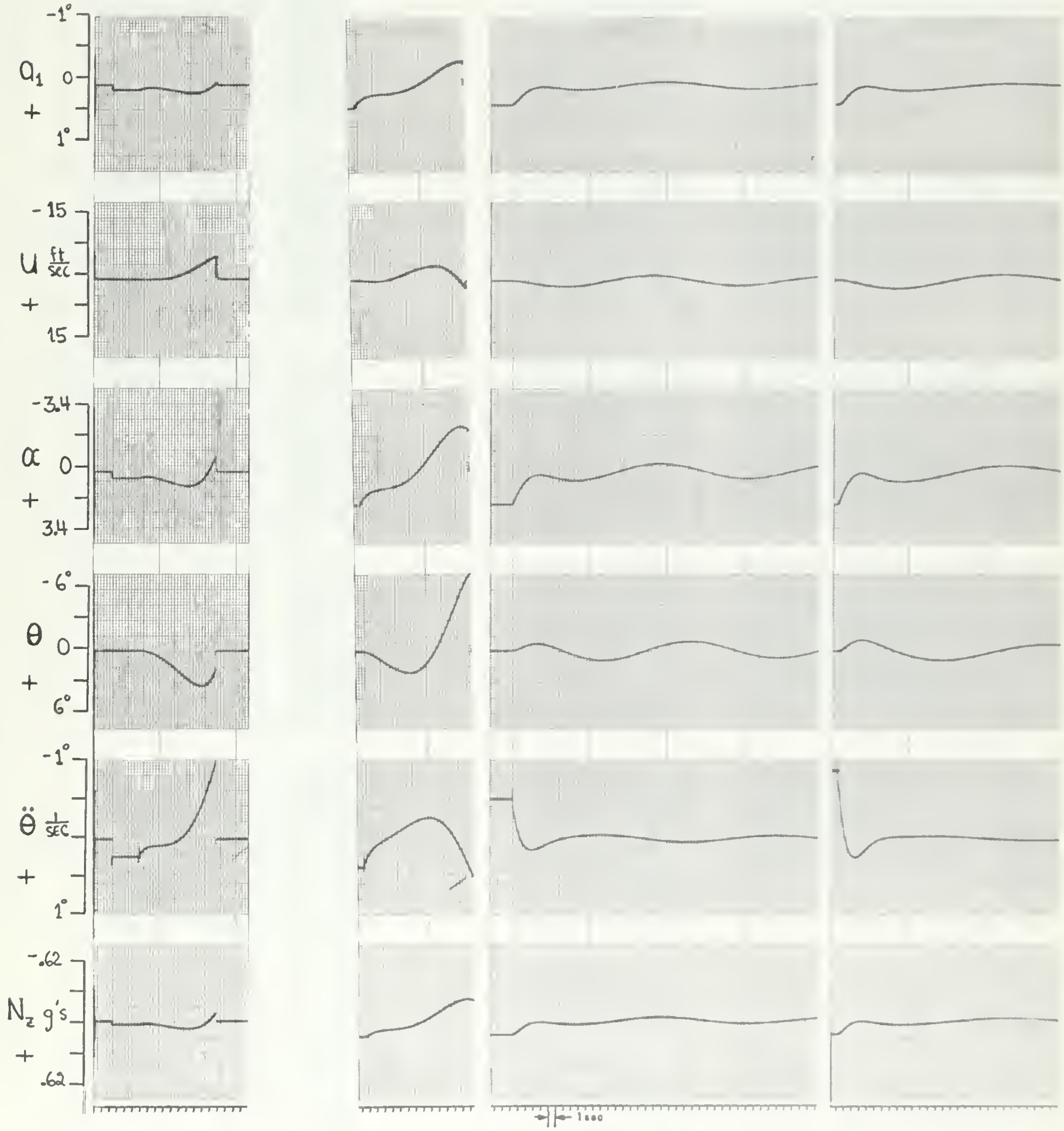
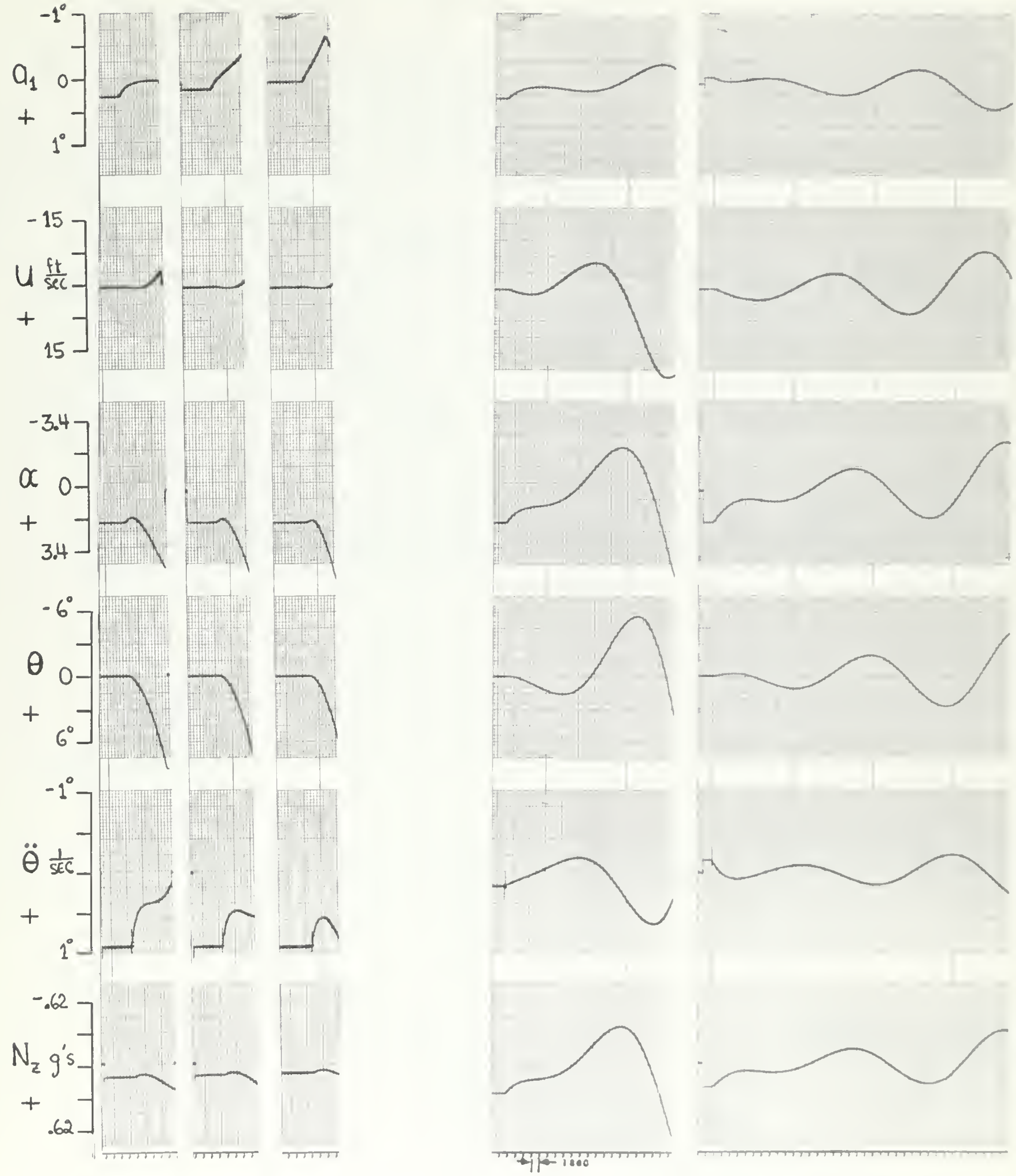


FIGURE (16): Longitudinal (Oehmichen) flapping feedback, $B_{1S} = K\ a_1$, for a) K = 0.5, varying cross control derivative, Z_{B_1} = .05, .25, b) Z_{B_1} = value of 42000 lb helicopter, varying K = 0.5, 1.0.



(a) (b)

FIGURE(17): Cyclic component of Delta Three feedback for configuration similar to, but not exactly, the 42000 lb helicopter, a) positive (conventional), K_δ = 0.1, b) negative, K_δ = -.25, -.75, -1.0.



(a)

(b)

FIGURE (18): Coning angle (collective flapping) feedback, $\theta_0 = -J\, a_0$, a) $M_{\theta_0} = 0$, J = .25, 0.5, 1.0. b) M_{θ_0} = value of 42000 lb helicopter, J = 0.5, 1.0.

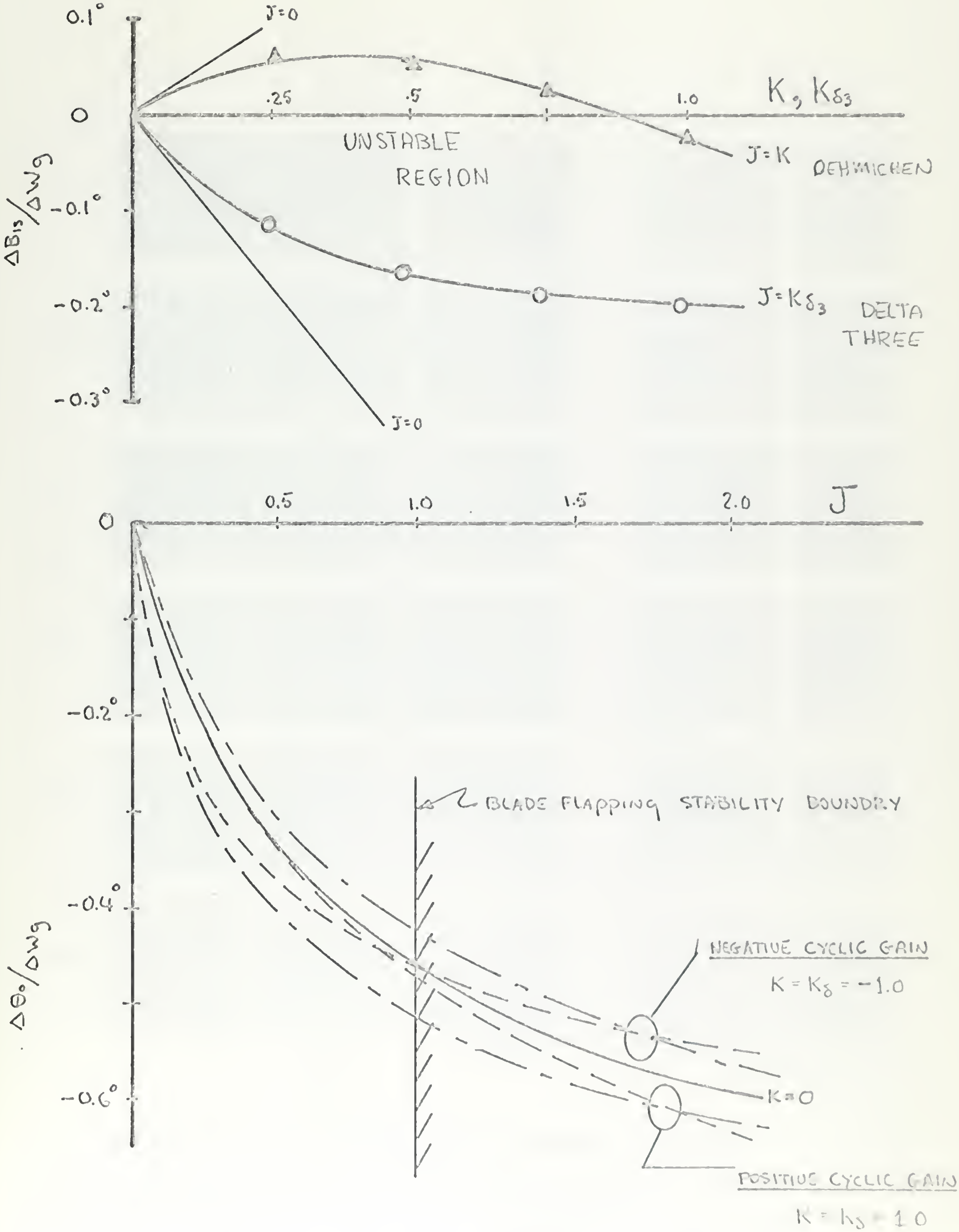


FIGURE (19): Static feedback elements: (top) cyclic feedback, (bottom) collective [illegible]

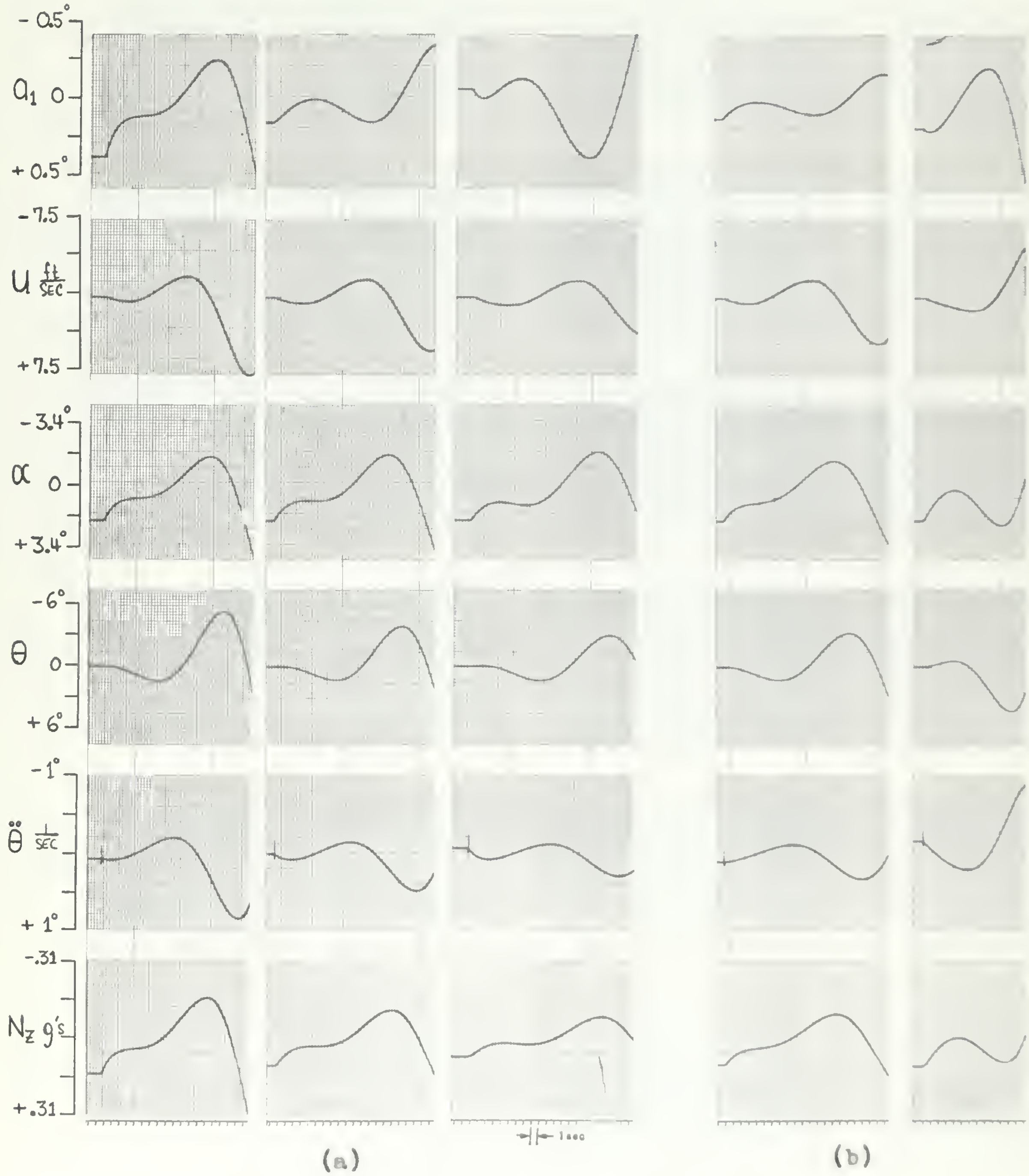


FIGURE (20). Oehmichen pitch flap coupling on 42000 lb helicopter, a) Effect of coning component gain, J = 0, 0.5, 1.0, @ K==0.5. b) Effect of cyclic component gain, K = .25, .75, @ J = 0.5.

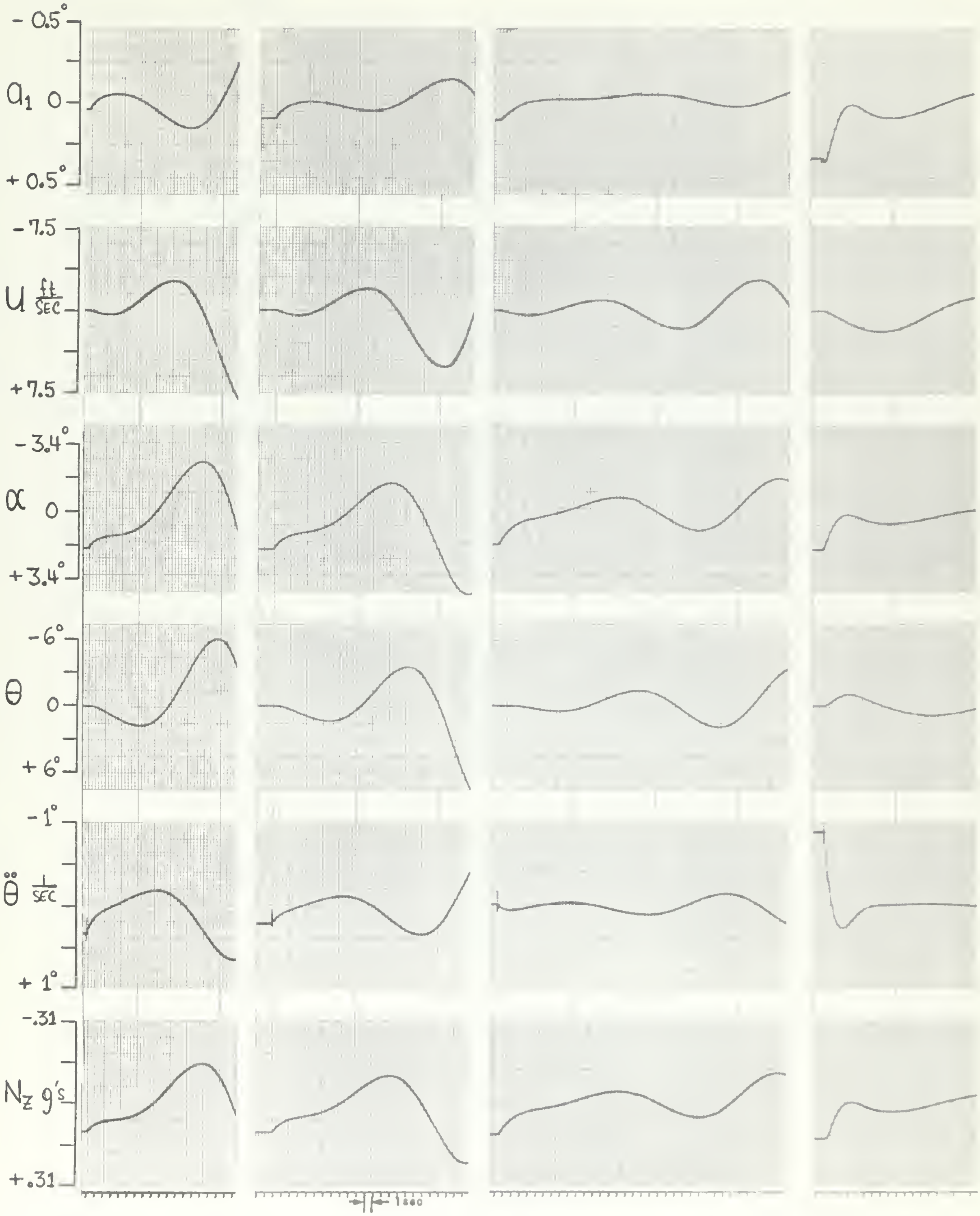


FIGURE (21): Delta Three pitch flap coupling, Effect of sign and magnitude of cyclic component on configuration similar to, but not exactly, the 42000 lb helicopter, K_δ = +.25, 0, -.25, -1.0, @ J=0.5.

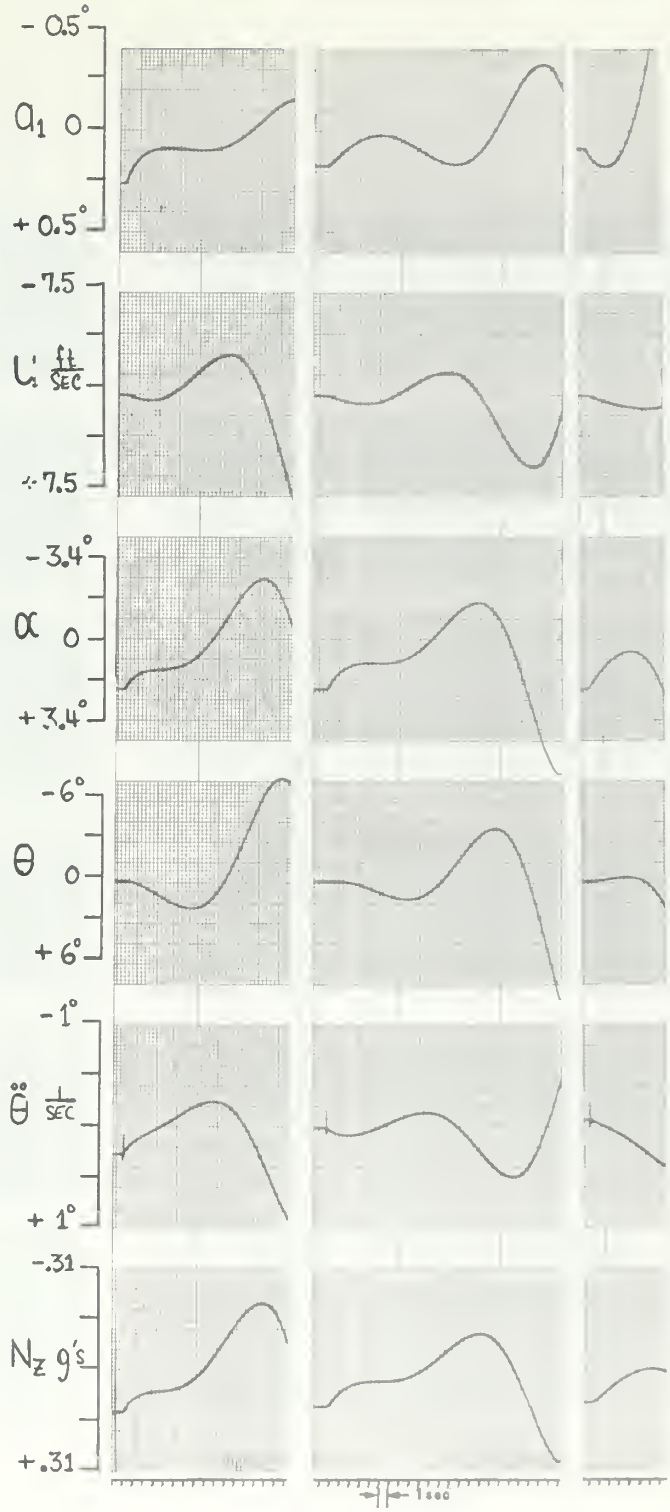
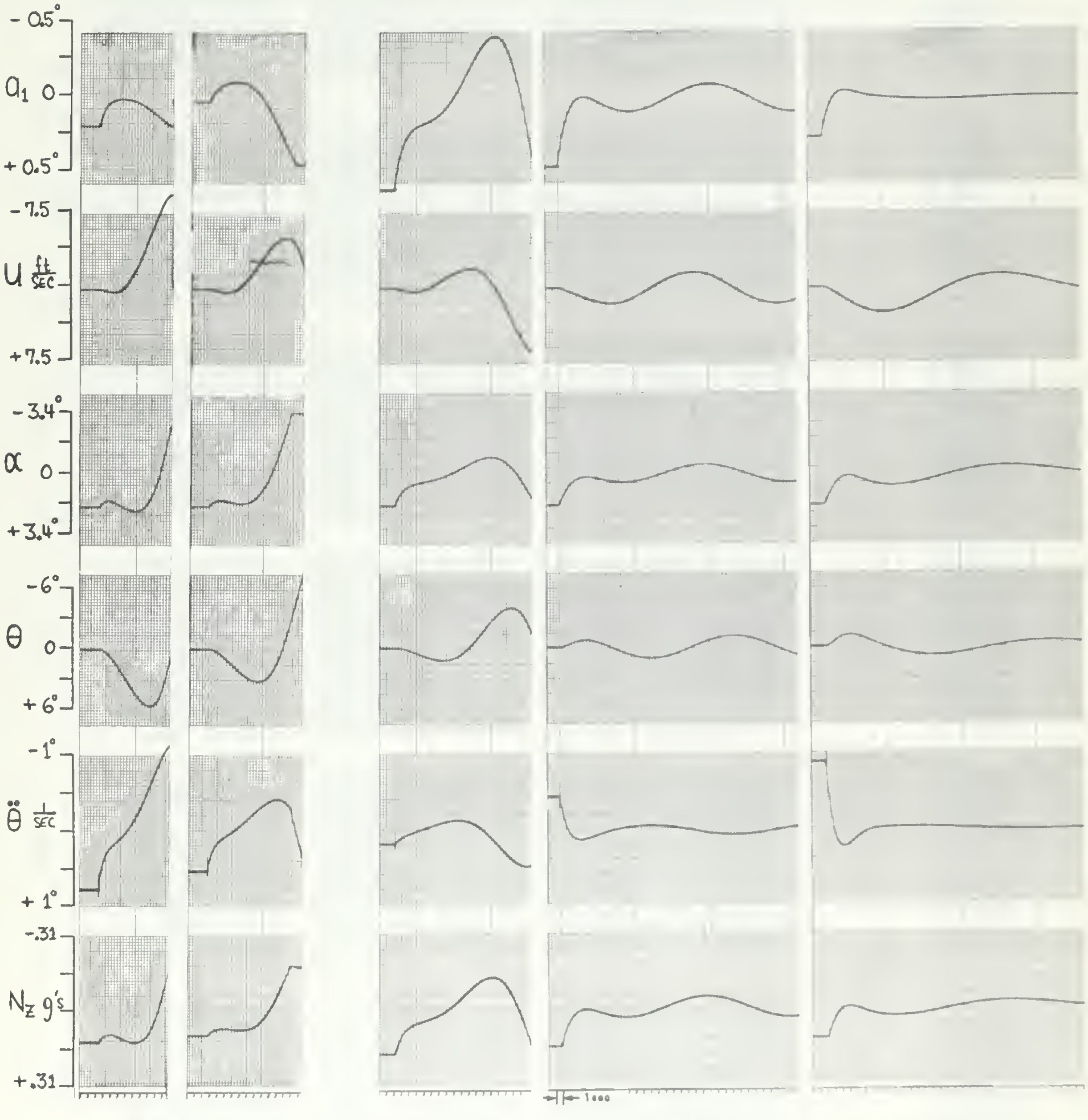


FIGURE (22): Oehmichen pitch flap coupling on 42000 lb helicopter; K = J = .25, 0.5, .75.



(a) (b)

FIGURE (23): Delta Three pitch flap coupling for configuration similar to, but not exactly, the 42000 lb helicopter. a) Positive (conventional) $J = K_\delta = .25$, 0.5 b) Negative $J = K_\delta = -0.5$, -.75, -1.0.

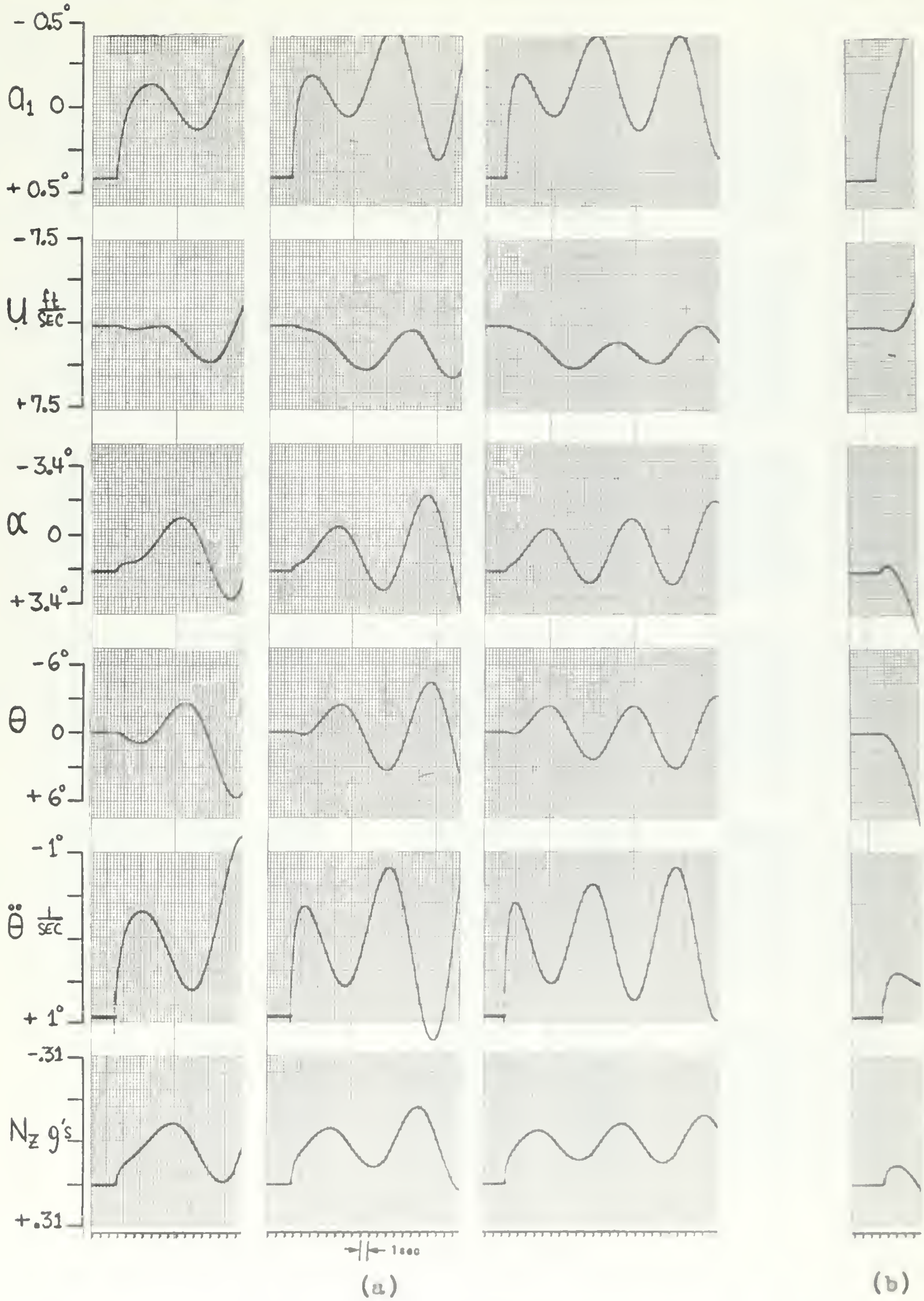


FIGURE (24): Integrated coning angle feedback, $\theta_o = -J\int a_o\, dt$,

a) M_{θ_o} = value of 42000 lb helicopter, J = .25, .75, 1.0,

b) M_{θ_o} = 0, J = .25.

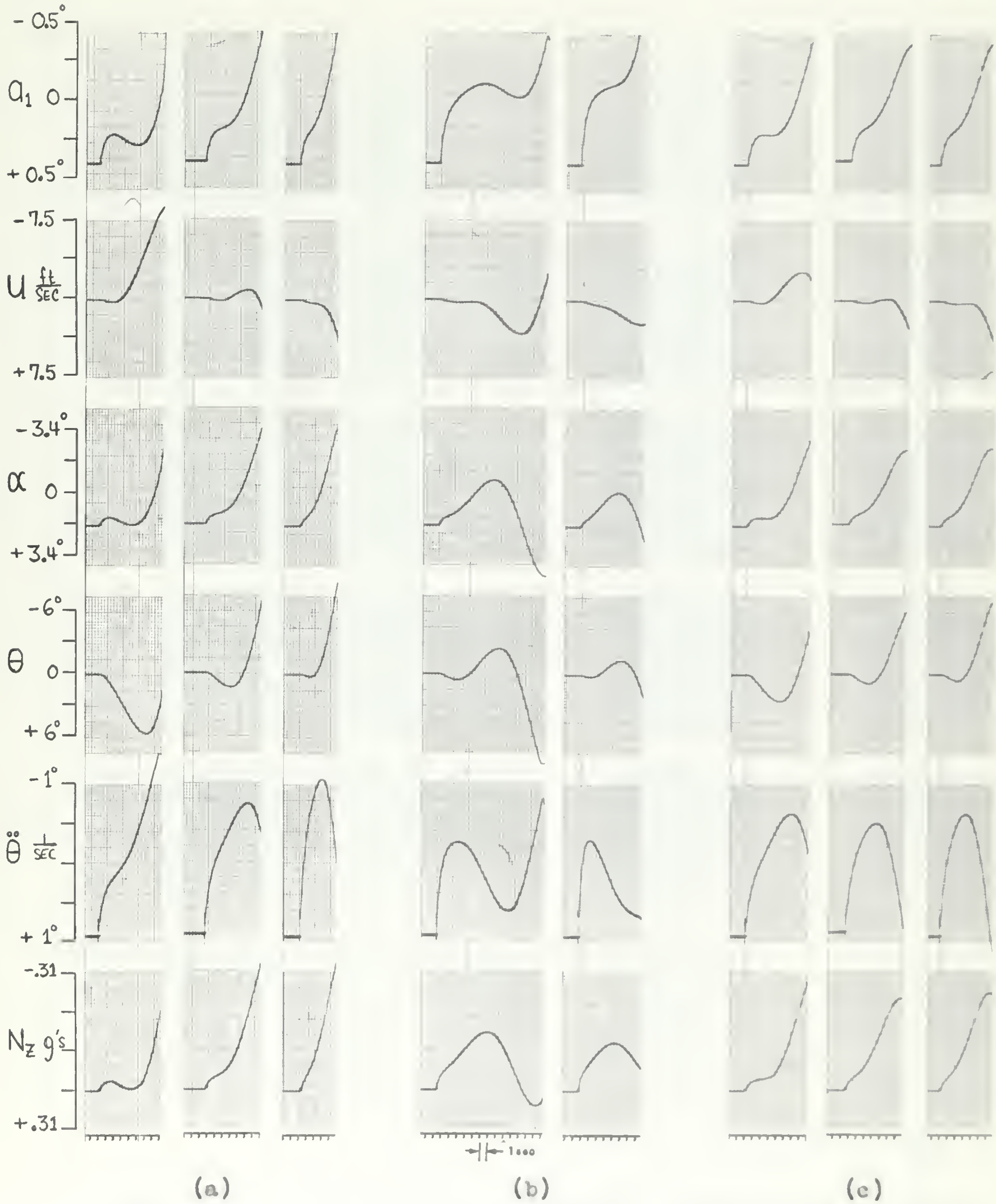
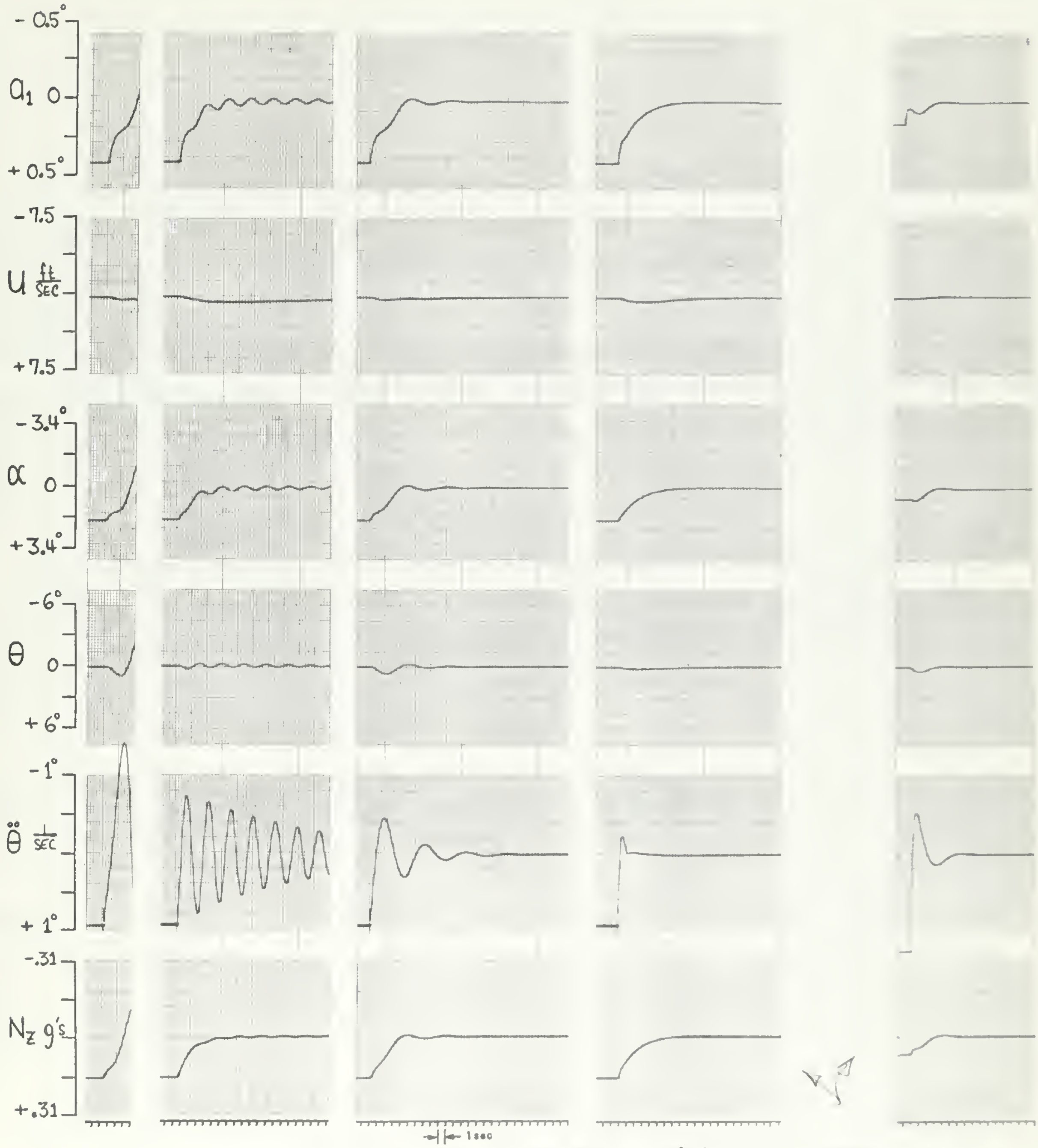


FIGURE (25): On the 42000 lb helicopter... a) Integrated cyclic component of Oehmichen feedback, $B_{1S} = K\int a_1$; K = .05, .25, .75. b) Integrated Oehmichen Feedback, $B_{1S} = K\int a_1\, dt$, $\theta_0 = -J\int a_0\, dt$, J = K = .25, 0.5. c) Idealized Lockheed Gyro, C_2 = 2/3. K_2 = 0.1, 0.4, 0.6.



(a) VALUES ON ACTUAL HELICOPTER (b)

FIGURE (26): Rate and Attitude feedback; $(\tau_1 s+1)\Delta B_{1s} = R(\tau_2 s+1)\Delta\theta_f$

a) 42000 lb helicopter, R = .281, 1) τ_1 = 0, τ_2 = .26, 2) τ_1 =0, τ_2 = 2.6, 3) τ_1 =.22, τ_2 = .26, 4) τ_1 = .22, τ_2 = 2.6, b) 18000 lb helicopter, R = .55, τ_1 = .1, τ_2 = .37.

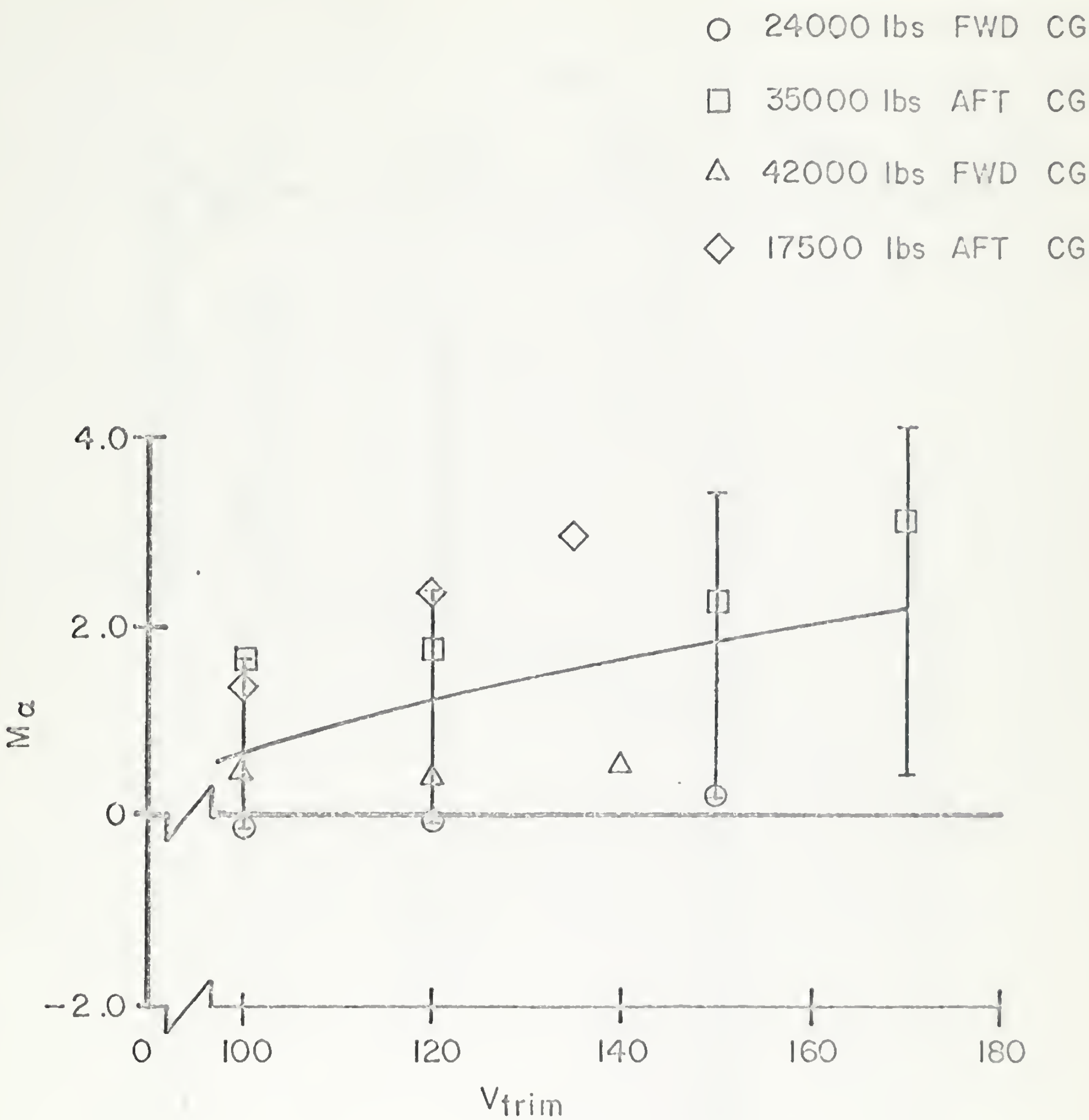
HELICOPTER DATA

The data contained in this appendix is taken from unpublished data on three large, high performance, single rotor helicopters. Forward and aft center of gravity positions, heavy and light gross weights, and three different configurations are represented. All of the helicopters have certain characteristics in common, and these characteristics are taken as representative of any heavy, high speed, single rotor, helicopter. They include large flapping hinge offset ($^e/R \doteq 5\%$), 5 and 6 bladed main rotors, high disk loading, and relatively large horizontal tails.

Some of the data did not cover the flight speed of interest in this work, so it was interpolated to extend 10 knots and 15 knots in two cases. The following table shows some of the pertinent information concerning the helicopters.

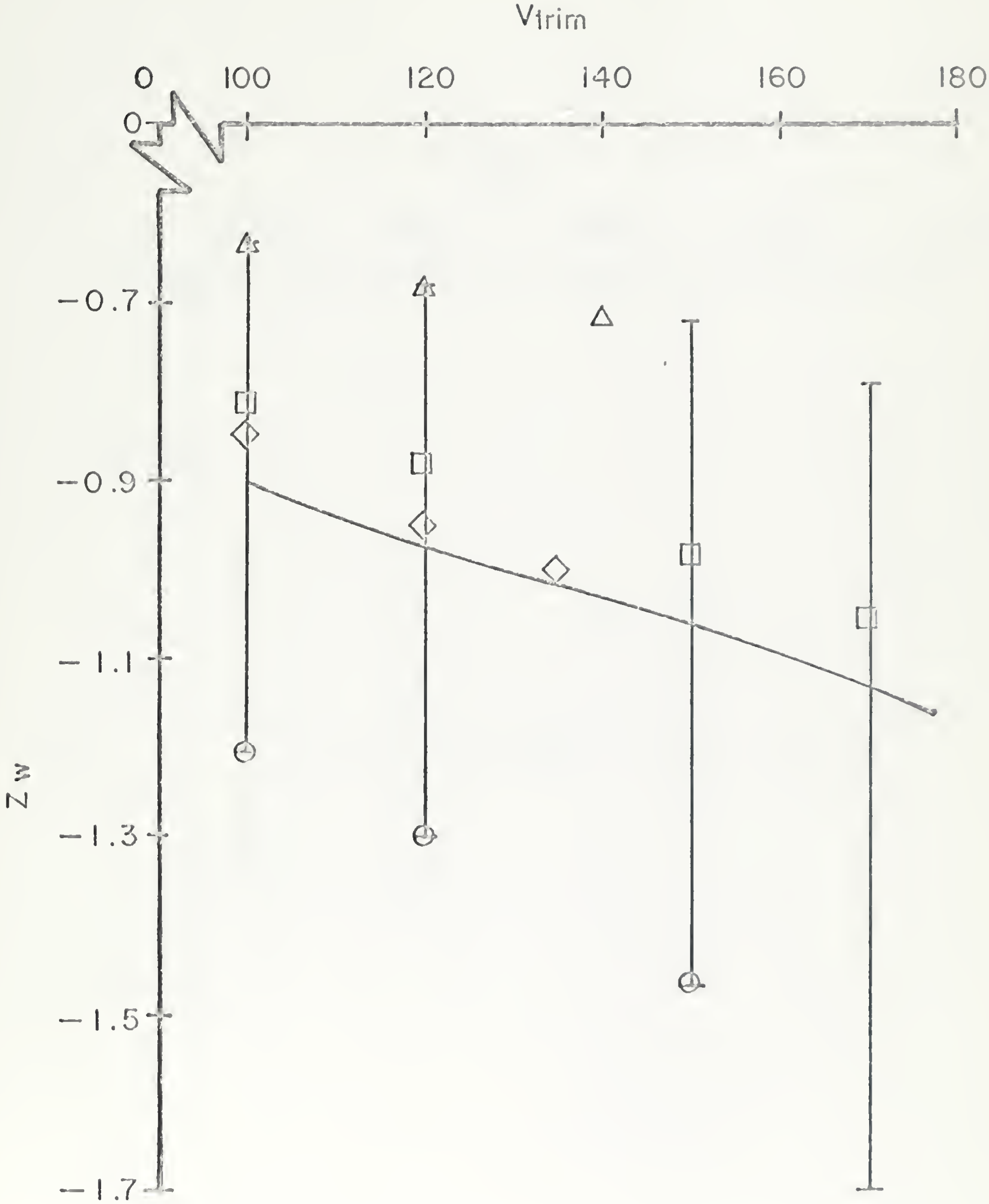
GROSS WEIGHT	C.G. LOCATION	MAX SPEED OF DATA (KTS)	TAIL SIZE($s_T\ l_T$)	$\frac{e}{R}$	$\frac{T}{A}$
42000	FWD.	140	1040	5.5%	9.3
35000	AFT.	170	1670	5.5%	8.2
24000	FWD.	150	1670	5.5%	8.2
18000	AFT.	135	955	3.2%	5.5

The data was reduced from a space fixed axis to a stability axis and dimensionalized. At each speed the high, low and average of each stability derivative was found and the information is presented in the figures. The lines thru the data points represent "typical", or "average" stability derivatives, and the trends of variation with forward speed.



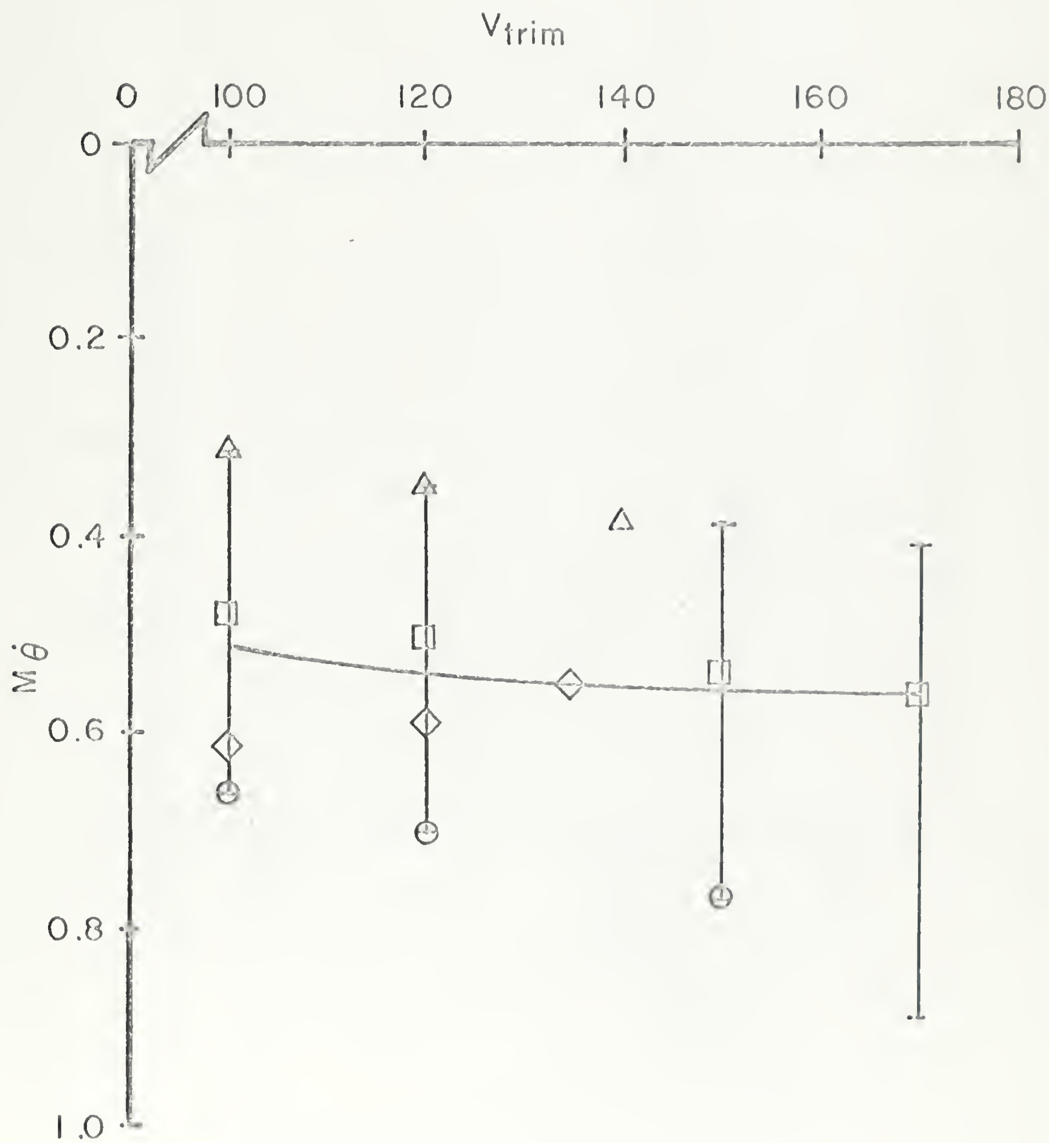
APPENDIX 1 (a)

M_α vs. U_o (kts)



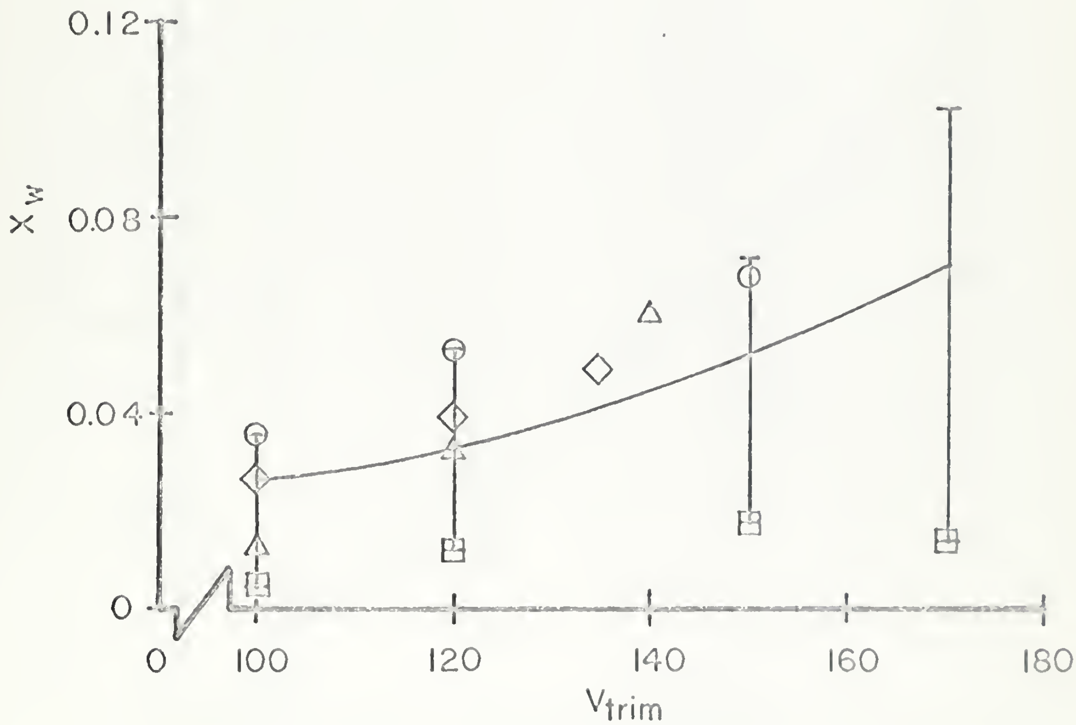
APPENDIX I (b)

Zw vs. U_o (kts)



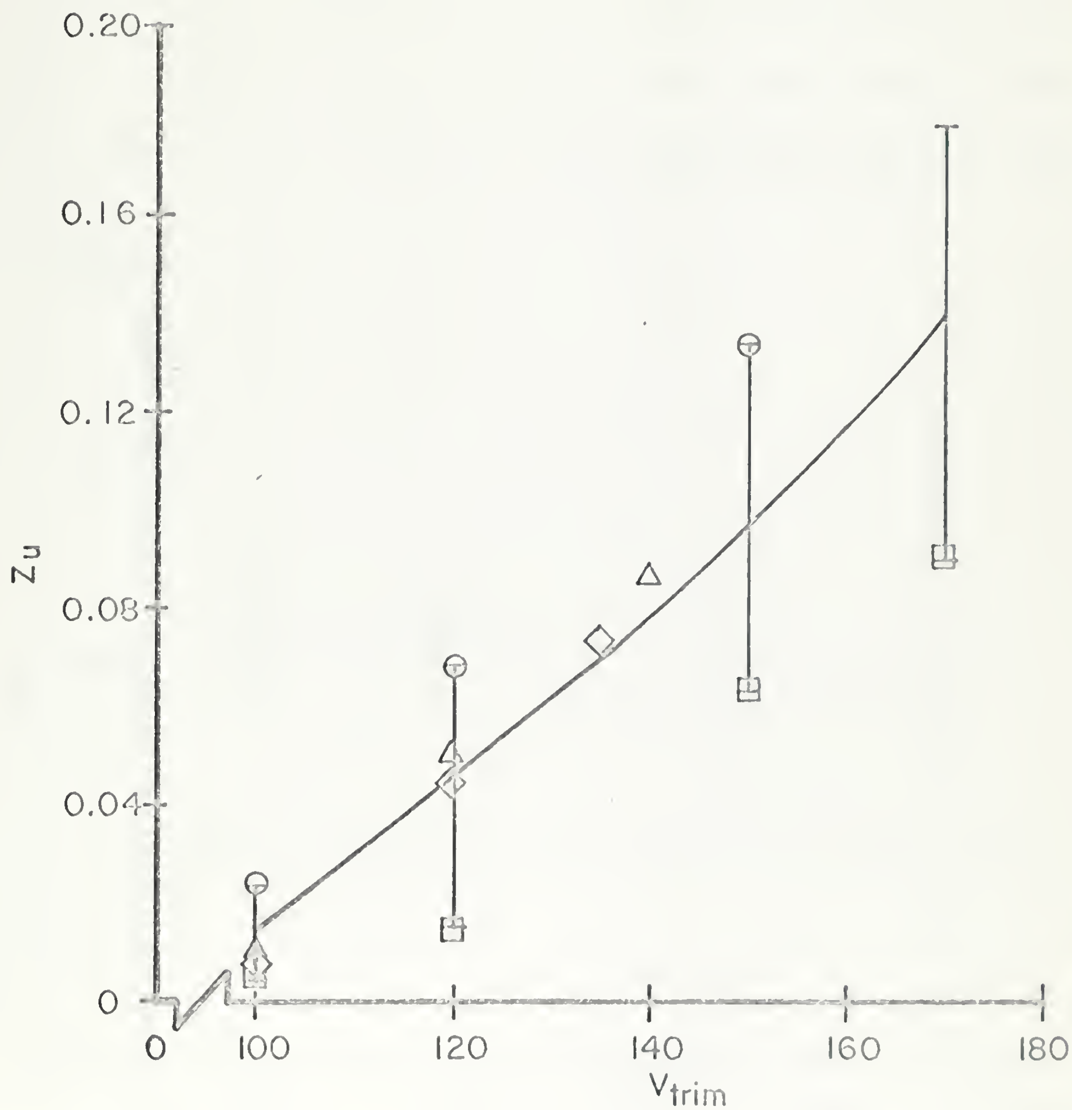
APPENDIX I (c)

$M_{\dot{\theta}}$ vs. U_o (kts)



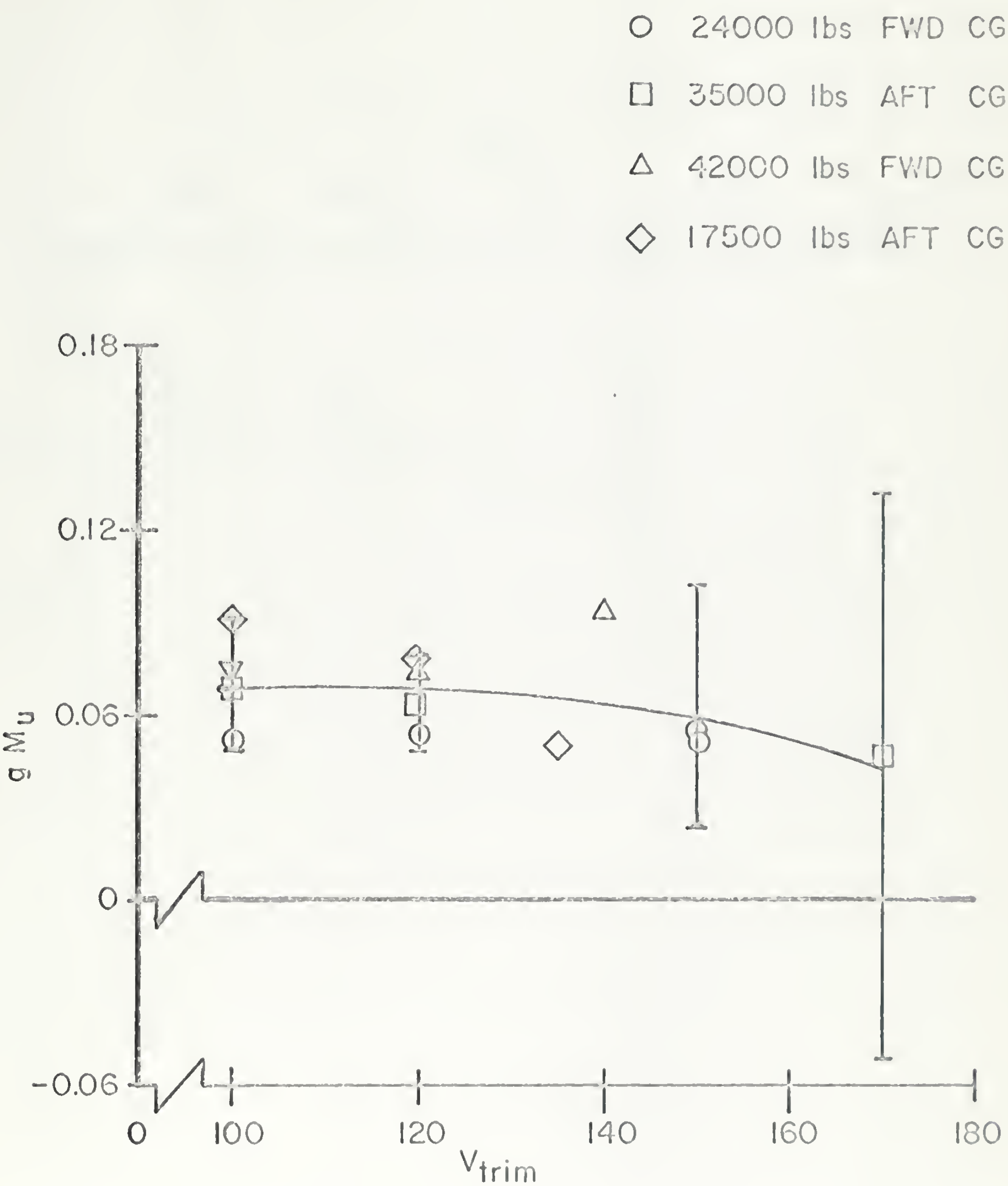
APPENDIX I (d)

Xw vs. U_o (kts)



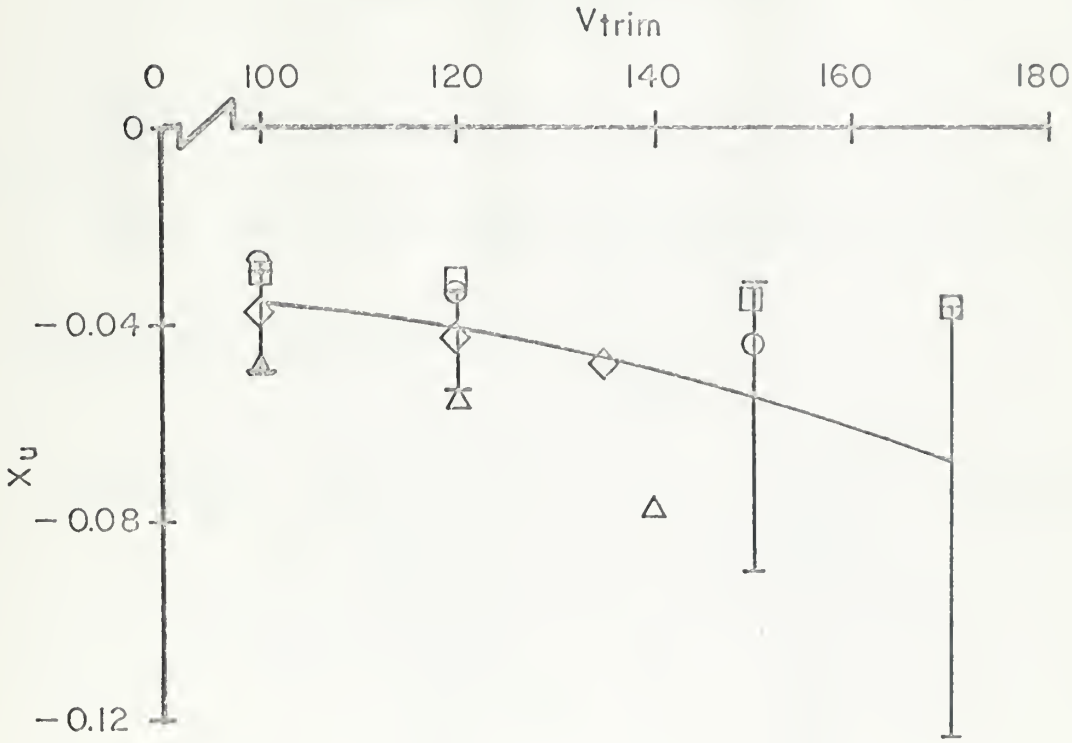
APPENDIX I (e)

Zu vs. U_o (kts)



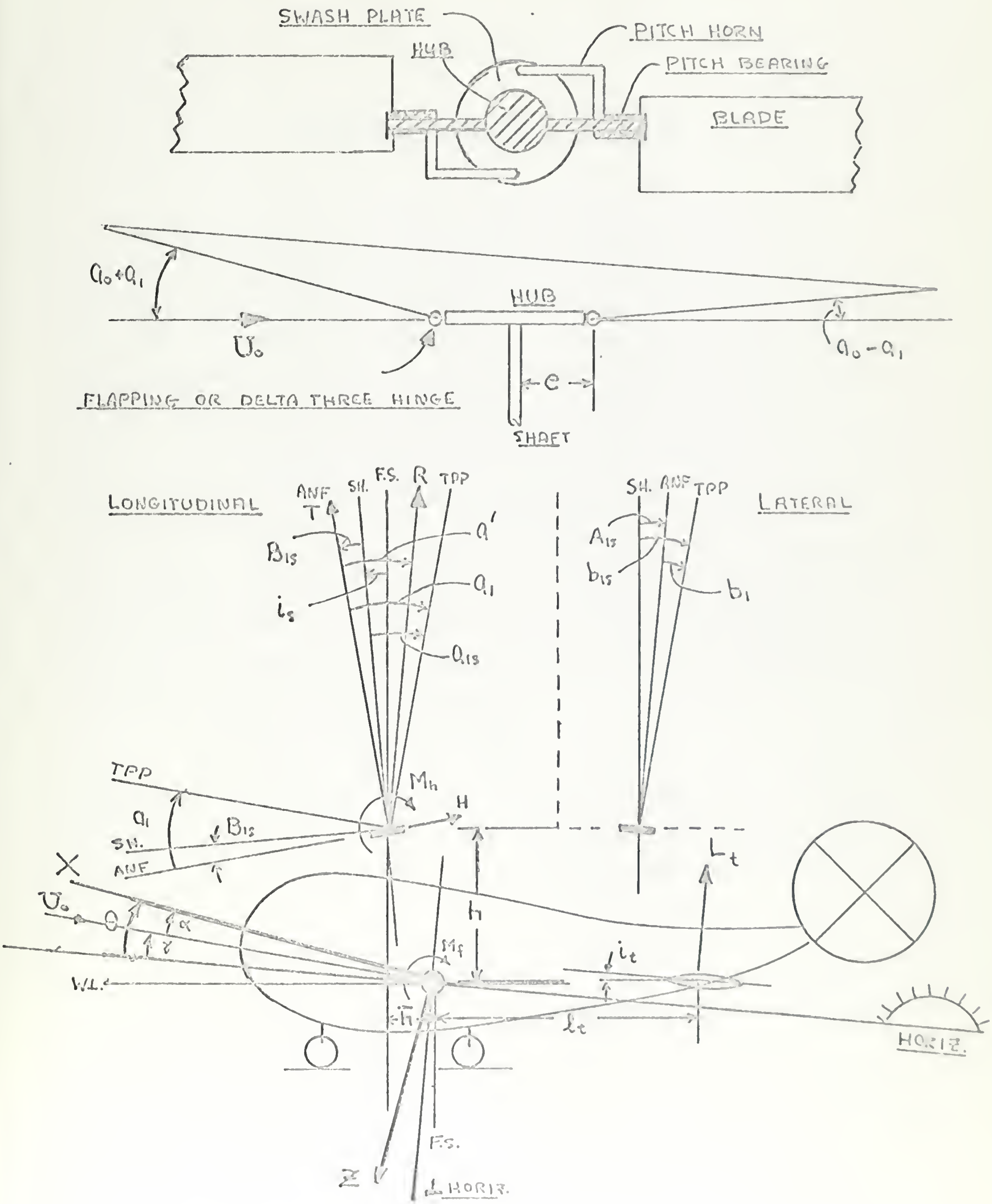
APPENDIX I (f)

gMu vs. U_o (kts)



APPENDIX I (g)

Xu vs. U_o (kts)



GEOMETRY, AXIS [illegible]

APPENDIX III

AXIS SYSTEMS AND AXIS CONVERSION

There are three types of axis systems commonly used in aircraft stability analyses, each having its own peculiar advantages. The conversion of data from one system to another is usually required when data comes from different sources or when the analyst is more familiar with one system. The axis systems are named by the orientation of the X axis. First, there is the horizontal, or earth fixed axis system. In this system the relative wind velocities are resolved into horizontal and vertical components. Second, there is a wind axis system in which the X axis is directed into the relative wind at all times. In both of the axis systems above, an aircraft attitude change (e.g. angle between wing chord and horizon) results in a new orientation of the X axis with respect to the aircraft. This changes the velocity components with respect to the aircraft, and moments and forces result. This results in non-zero stability derivatives like

$$\frac{\partial X}{\partial \theta}, \quad \frac{\partial M}{\partial \theta}, \quad \frac{\partial Z}{\partial \theta}$$

It is important to realize that these derivatives arise only because of the choice of axis system. Aerodynamic forces do not depend on pitch attitude of the aircraft body (e.g. wing chord) with respect to the horizon.

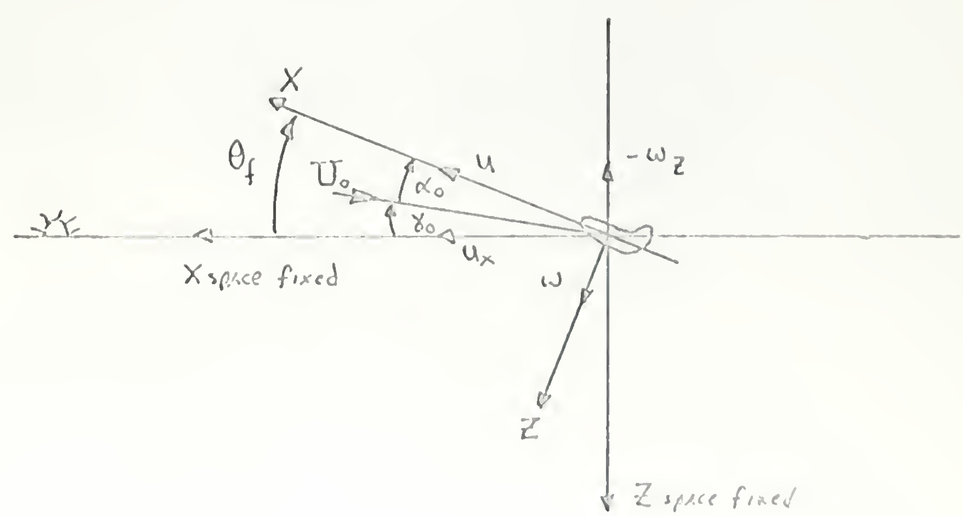
The one axis system left is the body axis system. As the name implies the X axis is fixed with respect to the body (e.g. a wing chord or a rotor shaft) and all velocities are resolved parallel and perpendicular to the body. With such a system an attitude change does not change the velocity components with respect to the body and hence no aerodynamic forces arise. This system is characterized by the absence of stability derivatives in θ, but results in a bit of confusion when trying to interpret physically the forces on the body.

This is because the rotating (in the case of a pitching or rolling rate) coordinate system introduces inertial, or centrifugal, forces in order to satisfy Newton's laws. Care must be taken to remember that these inertial forces arise only because the interpreter is in a fixed coordinate system and the aircraft in a rotating system. As such, the centrifugal forces are not really forces on the aircraft at all, but rather a way for the interpreter to satisfy himself that F = ma without having to switch to the rotating axes of the aircraft.

A special case of the body axis system is when the X axis is initially aligned with the relative wind, and then fixed to the body - all at time zero - but then stays fixed with that orientation in the body during aircraft motions. This further simplifies the equations of motion by eliminating a wo term in the Z force equation.

CONVERSION FROM SPACE FIXED TO STABILITY AXES

In one way of converting data from one axis system to another the velocity components in the first are resolved into the second and substituted into the equations of motion, expressed in the variables of the first. Regrouping results in equations of the same form but with different coefficients. These new coefficients are renamed the stability derivatives in the second axis system. This is demonstrated by the following example of converting from space fixed to stability axes.



From the diagram of the velocity vector components comes the relationship

$$u = u_x - w_z \Theta , \qquad w = w_z + u_x \Theta$$

where u, w are body axis velocities and u_x, w_z are horizontal (space fixed) velocities. Expanding, for small perturbations, gives

A (1)
$$\Delta u = \Delta u_x - (w_{z_0} \Delta\Theta + \Theta_0 \Delta w_z)$$
$$\Delta w = \Delta w_z + (U_{x_0} \Delta\Theta + \Theta_0 \Delta u_x)$$

The stability axes are defined by the initial angle of attack being zero so, for stability axes,

$$\Theta_0 = \alpha_0 + \gamma_0 = \gamma_0$$

If, in addition, initially straight level flight is considered then

$$\Theta_0 = \gamma_0 = 0$$

The velocity vector resolution, equation A (1), then becomes

A (2)
$$\Delta u \doteq \Delta u_x$$
$$\Delta w \doteq \Delta w_z + u_{x_0} \Delta\Theta = \Delta w_z + U_0 \Delta\Theta$$

The Z force equation may be written, in a space axis, as

$$\frac{\partial F_z}{\partial \dot{u}_x} \Delta\dot{u}_x + \frac{\partial F_z}{\partial u_x} \Delta u_x - \frac{\partial F_z}{\partial \dot{w}_z} \Delta w_z + \ldots = 0$$

Substituting the equations A (2) into the above,

$$\frac{\partial F_z}{\partial \dot{u}_x} \Delta \dot{u}_x + \frac{\partial F_z}{\partial u_x} \Delta u + \frac{\partial F_z}{\partial \omega_z} \Delta \mathring{\omega} + \frac{\partial F_z}{\partial \omega_z} \Delta \omega$$

$$+ \left[\frac{\partial F_z}{\partial \dot{\theta}} - \frac{\partial F_z}{\partial \dot{\alpha}_z} \right] \Delta \mathring{\theta} + \left[\left.\frac{\partial F_z}{\partial \theta}\right|_{\alpha_z} - \left.\frac{\partial F_z}{\partial \alpha_z}\right|_{\theta} \right]_{u_x} \Delta \theta = 0$$

The first bracket term reduces to

$$\frac{\partial F_z}{\partial \dot{\gamma}} \Delta \mathring{\theta} = -U_o m \Delta \dot{\theta}$$

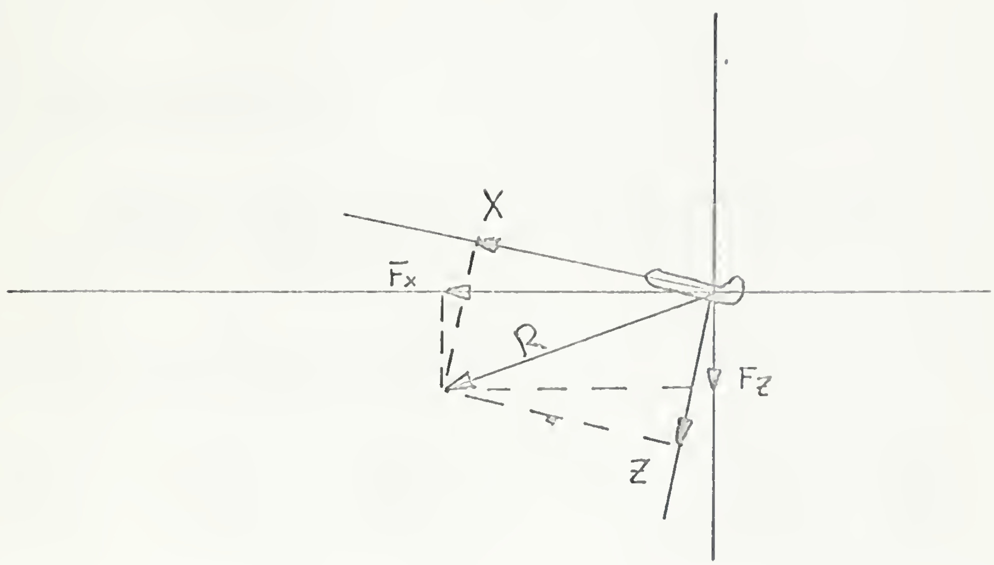
which is the centrifugal force that arises due to the change in axis systems (from stationary to rotating). The second bracketed term is identically zero (this is emphasized by the indication of what variables are held constant when those partial derivatives were taken). The other coefficients are now renamed in the body axis as (after dividing them by m)

A (4) $$Z_{\dot{u}} \Delta \dot{u} + Z_u \Delta u + Z_{\dot{\alpha}} \Delta \dot{\alpha} + Z_\alpha \Delta \alpha - U_o \Delta \dot{\theta} = 0$$

The terms $Z_{\dot{u}}$ and $Z_{\dot{\alpha}}$ are due to unsteady aerodynamics and are zero in this work.

Another method that can be used to convert from one axis to another may be more straight forward and can provide more insight into the nature of the two axis systems. From the force balance diagram below come the expressions for body forces in terms of horizontal forces. (The moments are equal in both systems)

$$X = F_x - F_z \theta$$
$$Z = F_z + F_x \theta$$
$$M = M$$



Each body axis derivative is then found by applying the chain rule as is demonstrated for Xu below.

$$\left.\frac{\partial X}{\partial u}\right|_{w\theta} = \frac{\partial F_x}{\partial u_x}\frac{\partial u_x}{\partial u} - \frac{\partial F_z}{\partial u_x}\frac{\partial u_x}{\partial u}(\theta_o) - F_{z_o}\frac{\partial \theta}{\partial u_x}\frac{\partial u_x}{\partial u}$$

$$+ \frac{\partial F_x}{\partial u_z}\frac{\partial u_z}{\partial u} - \frac{\partial F_z}{\partial u_z}\frac{\partial u_z}{\partial u}(\theta_o) - F_{z_o}\frac{\partial \theta}{\partial u_z}\frac{\partial u_z}{\partial u}$$

From equation A (1) comes

$$\frac{\partial u_x}{\partial u} \doteq 1 \qquad \frac{\partial u_x}{\partial w} \doteq \theta_0$$

$$\frac{\partial u_z}{\partial w} \doteq 1 \qquad \frac{\partial u_z}{\partial u} \doteq -\theta_0$$

And in the equation A (5)

$$\frac{\partial \theta_0}{\partial u_x} = \frac{\partial \theta}{\partial u_z} = 0 \quad , \quad \theta_0 = \gamma_0 + \alpha_0$$

Rewriting equation A (5) for any body axes

$$\frac{\partial X}{\partial u} = \frac{\partial F_x}{\partial u_x} - \frac{\partial F_z}{\partial u_x}\alpha_0 + \gamma_0 \left\{ \theta_0 \frac{\partial F_z}{\partial u_z} - \frac{\partial F_x}{\partial u_x} - \frac{\partial F_z}{\partial u_x} \right\}$$

Finally, for stability axes

$$\gamma_0 = \theta_0$$

and

$$\frac{\partial X}{\partial u} = \frac{\partial F_x}{\partial u_x} + \gamma_0^2 \frac{\partial F_z}{\partial u_z}$$

A (6)

$$- \gamma_0 \left\{ \frac{\partial F_x}{\partial u_x} + \frac{\partial F_z}{\partial u_x} \right\}$$

The same procedure is carried through for every derivative in the body axis equations of motion. The result of all the conversion is:

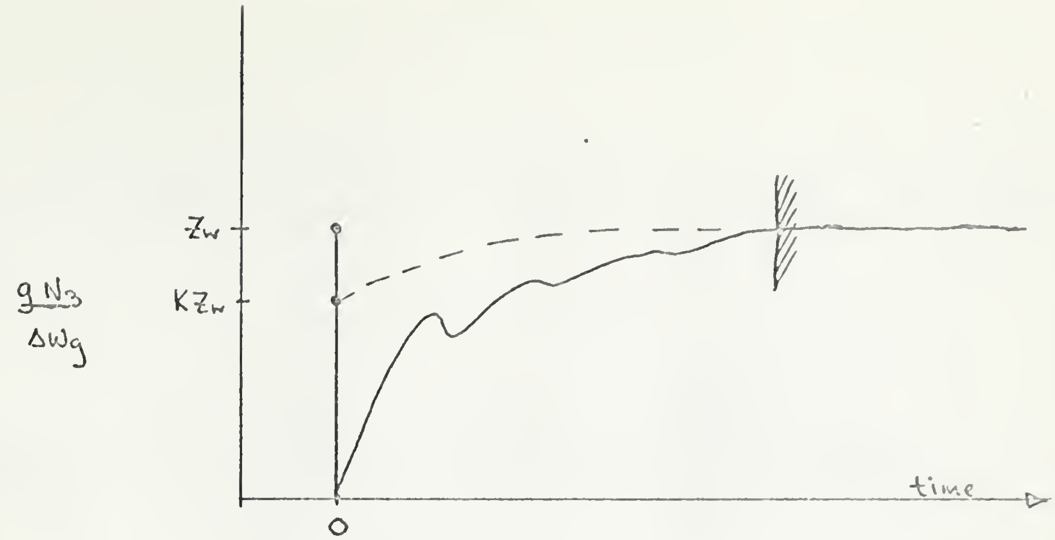
STABILITY AXIS		SPACE AXIS
Xu	$\doteq$	$Fxu_{x} - \gamma_o [Fxu_{z} - Fzu_{x} - \gamma_o Fzu_{z}]$
Xw	$\doteq$	$Fxu_{z} - \gamma_o [Fzu_{z} - Fxu_{x} + \gamma_o Fzu_{x}]$
Zu	$\doteq$	$Fzu_{x} + \gamma_o [Fxu_{x} - Fzu_{z} - \gamma_o Fxu_{z}]$
Zw	$\doteq$	$Fzu_{z} + \gamma_o [Fzu_{x} + Fxu_{z} + \gamma_o Fxu_{x}]$
Mu	$\doteq$	$Mu_{x} - \gamma_o Mu_{z}$
Mw	$\doteq$	$Mw_{z} + \gamma_o Mu_{x}$
$M\dot{\theta}$	$\doteq$	$M\dot{\theta}$

APPENDIX IV

GUST ALLEVIATION

For the simple aircraft of this work the initial (at time = 0^+) load factor is instantaneous and proportional to the gust magnitude and Zw. In actuality, for the real helicopter or airplane, this is not the observed response. The differences between the actual and the model responses are attributed to and defined herein as gust alleviation. The definition used here is in contrast to another definition that would interpret this whole work as a study of means by which to alleviate the magnitude of a helicopter's response to gusts, and thus considers artificial stabilization as gust alleviation. Many parameters may influence the magnitude and dynamics of gust alleviation. In reference (8), for example, is shown the derivation for a gust alleviation factor, or a corrective factor to multiply times the static value of Zw, which should be used to compute the initial load factor for a static model (e.g. rotor bolted down in a wind tunnel) due to a more complicated inclusion of the rotor downwash. In reference (5) is shown the result of a detailed computer program used to compute gust alleviation factors for the initial and the dynamic response by including unsteady aerodynamics and aero-elasticity to compute an effective value of Zw that varies with time and the motion of the helicopter. A very detailed computer analysis such as that used in reference (5), however, may or may not have included every cause of gust alleviation, and it is hard to gather any insight as to which are the important causes and their direct effects. The only conclusion that can really be drawn is that, for helicopters encountering sharp edged gusts, the effective value of Zw is dynamic and may vary from the static value.

In the figure below are shown examples of what load factor responses could look like for the static, zero degrees of freedom model if gust alleviation were considered. (Note: these are not computed or predicted responses but are drawn for illustration only).

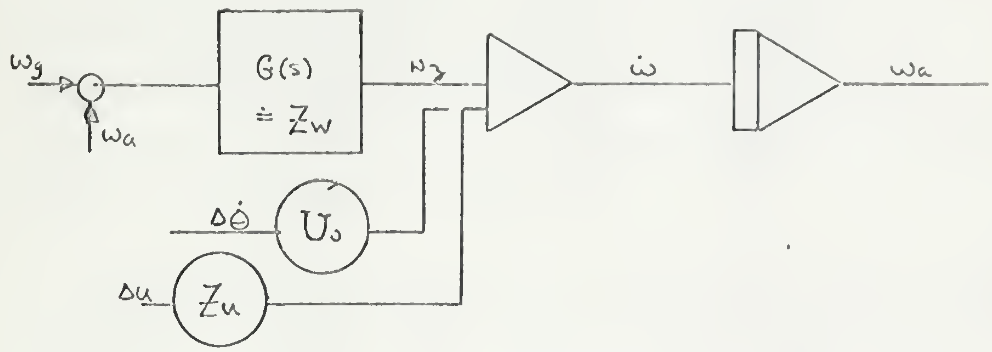


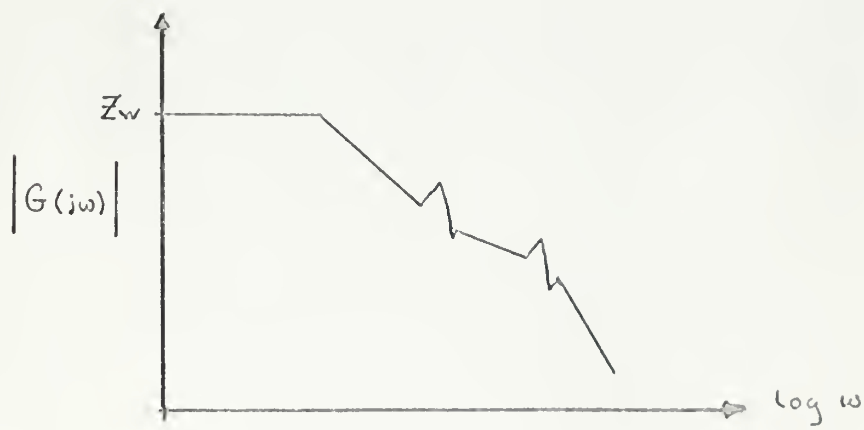
The value at time = 0^+ is shown as Zw for the linear model, and KZw for the kind of static analysis presented in reference (8). The dotted line is descriptive of the results that would be obtained by extending the static analysis in an iterative manner - computing the thrust at each increment in time, finding a corresponding downwash velocity and using that to compute the thrust at the next point, and so on. The remaining line illustrates the kind of dynamic response that might be found to ensue with an analysis such as in reference (5). In accord with assumptions concerning a quasi-static analysis, and neglecting unsteady aerodynamics and rotor plane dynamics in this work, a constant linear value of Zw is used corresponding to the value to the right of the hatched area on the figure.

In the static model the only aerodynamic variable that the aircraft responds to is Δw_g because $\Delta\theta = \Delta w_a = 0$.

As degrees of freedom are added the motions of the aircraft couple with the gust velocities to cause aerodynamic variables that are functions of time and the aircraft motions. As the frequency of the motions increase from zero (as in the static case) to the full three degrees of freedom motions, the errors of neglecting alleviation increase and become obscured. The same is true if the frequency of the gust disturbances is increased.

It is helpful to think of gust alleviation as a dynamic transfer function to replace the Zw and Mw potentiometers on an analog computer.





At very low frequency the transfer function reduces to Zw, as the Bode diagram shows, but at a higher frequency there may be a dynamic response.

Two ways of conducting an analysis are contrasted here: A detailed analysis in which every conceivable cause and effect is included for analysis, and a simple analysis in which only the most important, or first order, effects are considered. This study is of the latter classification. Until the first order, important causes of alleviation can be determined and modeled simply, however, there is little justification for the inclusion of any alleviation in the simple study. Other investigations, including flight testing, must be performed to determine which are the first order causes and what are their important effects. The scope of this paper does not include such investigation.

A pertinent study would be to compare the results of simple analyses such as this with full scale flight test gust response data. This could lead to some insight as to how alleviation really influences the dynamics as a whole. It might then be instructive to construct different filters on the analog computer and put them between the gust inputs and the summing amplifiers to try to reproduce the flight test data. The characteristics of a particular filter that did the job might give a clue as to what the real causes are.